Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about
Volume 3, Issue 8 (2025)

Online ISSN 3007-3197
Print ISSN 3007-3189

http://amresearchreview.com/index.php/Journal/about

Interdisciplinary Applications of Applied Mathematics and Partial Differential Equations in Modeling Complex Multi-Scale Phenomena

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ABSTRACT

Background: In disciplines involving the modeling of complex phenomena of multiple scales, there is a need to have robust models that have the potential of capturing the interaction at multiple spatial and temporal domain. PDEs provide an effective solution to model a dynamic system, and the combination with applied mathematics yielded remarkable contributions across sectors like biomedical engineering and environmental science to computational modeling. **Objective**: This paper set out to understand how applied mathematics and PDEs are used interdisciplinarity in modelling of complex systems with many scales of lower focus on existing trends, modelling strategies, challenges, and computational methods within a wide range of sciences. **Method**: To reach the conclusion, a qualitative systematic review of peer-reviewed articles has been carried out in database Scopus, Web of Science, and ScienceDirect published between 2021 and 2025. Thirty studies of high caliber were chosen depending on relevance and methodological superiority. Thematic analysis was done to stratify the PDEs according to types, their uses in domains, intended modeling, representation in scale and what computing strategies. **Results**: The PDE app area with the largest percentage of applications (30 percent) was in biomedical sciences followed by environmental modelling and material sciences. Algebraic Navier-Stokes and reaction-diffusion equations were used in the greatest frequency. The most important modeling can be done to have simulation,

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Volume 3, Issue 8 (2025)

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Print ISSN 3007-3189

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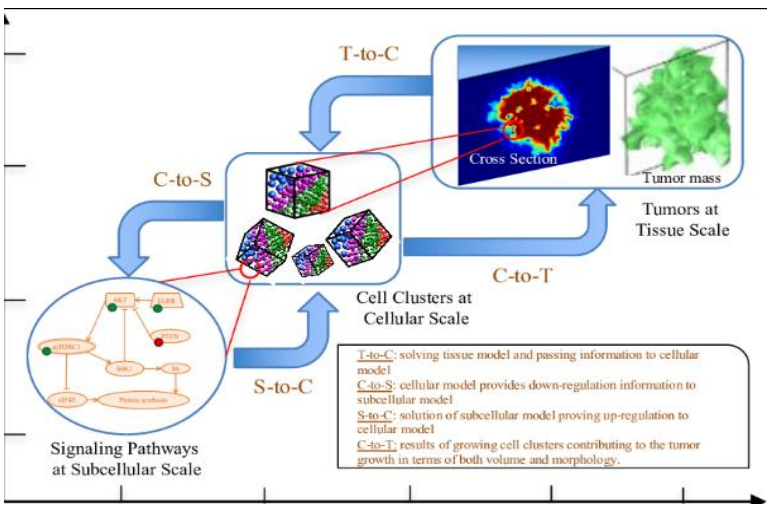
optimization and pattern formation. It was noted that such obstacles as computer cost, scale bridging and data sparsity can be major. There was the appearance of the hybrid approaches that combine PDEs with machine learning (e.g., PINNs) as a good solution. **Conclusion**: PDE-based modeling paradigm is an essential interdisciplinary paradigm that is used in modeling complex systems on different scales. Its growing synchronization with data-driven techniques heralds a major transition to more adaptive and predictive elucidation frameworks of science.

Keywords: Partial Differential Equations, Multi-Scale Modeling, Applied Mathematics, Interdisciplinary Modeling, Simulation, Hybrid Computation, Reaction-Diffusion Systems, Navier-Stokes Equations.

Introduction

The close connection between applied mathematics and the science of modeling in the past few decades has developed into a highly potent cross dimensional mode of analyzing natural and manufactured systems. Partial differential equations (PDEs) are some of the most useful mathematical tools that can be used to model space time dynamic processes. PDEs provide a mathematically structured method of representing change and continuity in physical phenomena, biological phenomena, social phenomena, and are therefore dominating the mathematical text depicting multi-scale systems. Through them, the fundamental principles of conservation, diffusion, transport, and the rates of reactions are captured on which the computational simulations, informing theory and practice, are based (Evans & Gariepy, 2021).

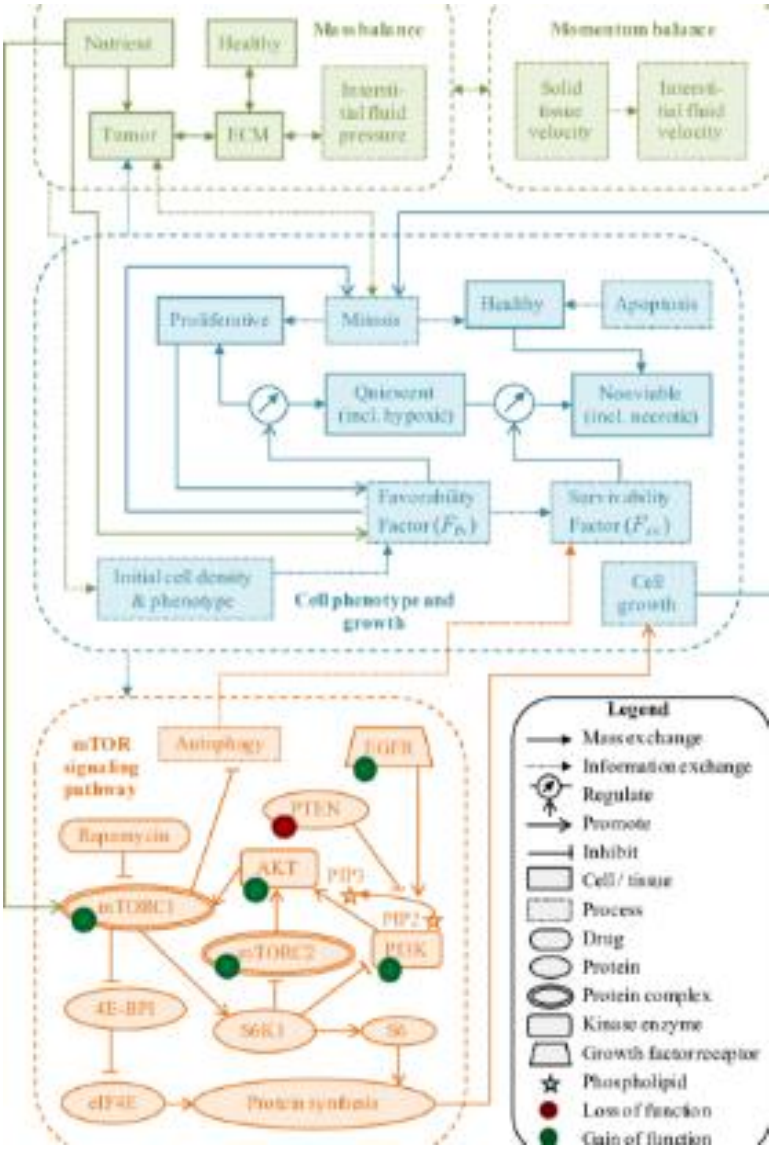
Complex systems are often embroiled with interactions at many different spatial-temporal scales, with small-scale behavior at the micro scale shaping macro-scale dynamics and vice versa. Examples cut across



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fields, including fluid dynamics in weather and blood flow in human vessels as well as quantum behavior in nano-structure and pattern development in ecosystems. Mathematical modeling of such a multi-scale system needs to be integrative in that, mathematical abstraction is interfaced with an empirical verification. Sufficiently well-behaved discrete, noisy data can be transformed using applied mathematics via PDEs into continuous data whose simulation, analysis, and predictive application can be used (Oden et al., 2022). Such models are frequently generalized with a combination of deterministic PDEs and stochastic elements to describe those uncertainties which exist in a biological or socio-economic setting.



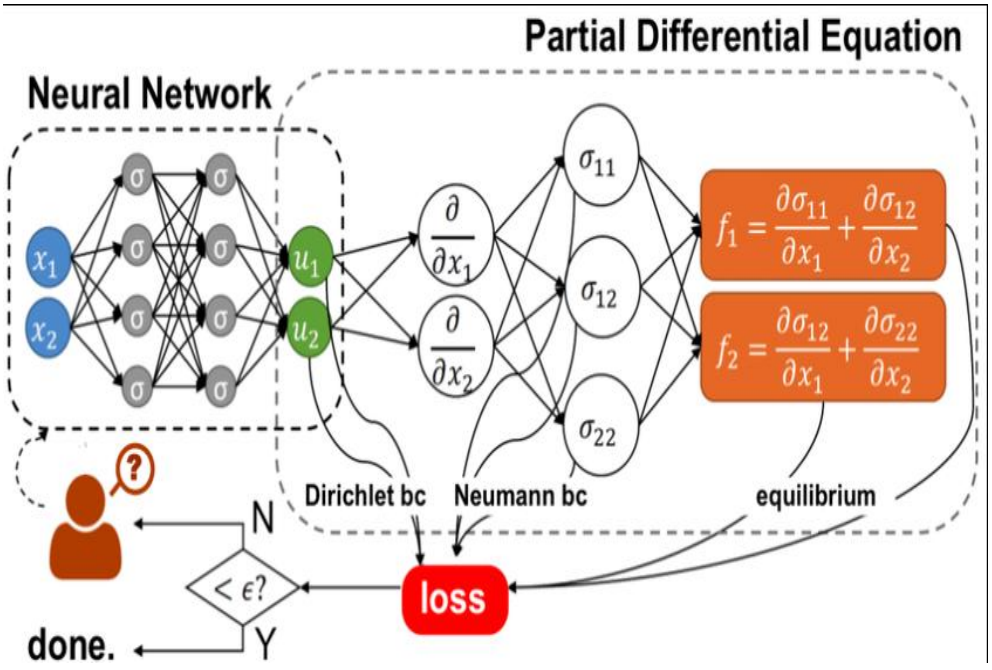
Generally, in the biomedical sciences, PDEs have transformed how we perceive the growth of tissues, cancer progression, and drug delivery. The spatial distribution and transmission of cellular signals via reaction-diffusion systems and intra-vascular movement and transport of therapeutic agents via convection-diffusion equations are modeled (Geris et al., 2021). Using the multi-scale modeling approach enables the scientists to approach the interactions at molecular level, all the way up to the organism functionality. As an example,

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multiscale cardiac models - an integration of electrophysiological equations and tissue level dynamics are used that can simulate arrhythmia and apply in the clinical intervention. This is by way of such applications that show that application of mathematics has moved beyond theoretical knowledge and has become a vitally important facet in translational science.

Other applications of PDEs are in environmental studies, on climate systems, ocean currents and groundwater contamination. Climate models that predict global warming or dynamics of natural disasters are based on the Navier-Stokes equations with addition of thermodynamic and chemical reaction terms. They are solved over a wide range of scales, including local atmospheric turbulence and planetary circulation patterns, and hence need an adaptive numerical methodology and advanced computing (Stevens et al., 2023). These applications are interdisciplinary in that they require the interaction of atmospheric physics, computational fluid dynamics and data science, which require new hybrid approaches to characterize the complexity of scale bridging.



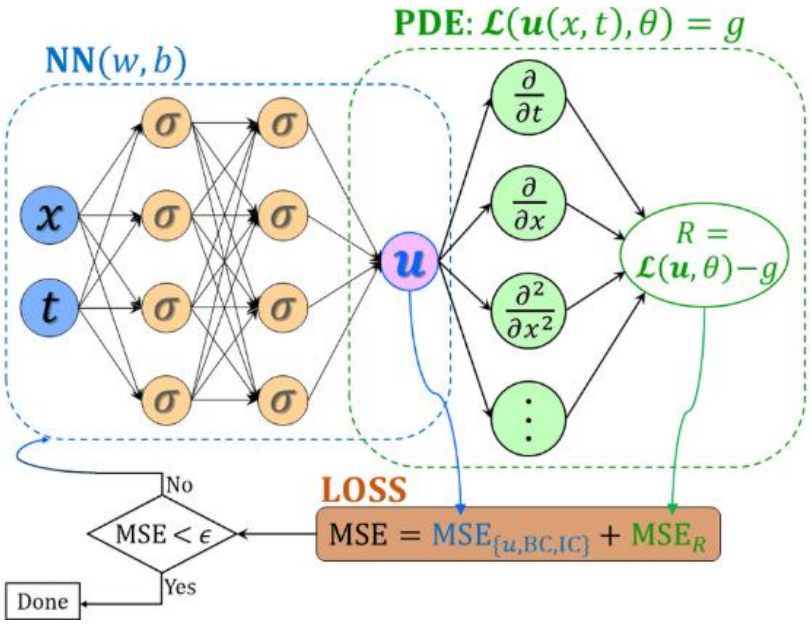
PDEs in materials science and engineering describe the phase transitions and mechanical response of complicated composites, such as polymers, alloys and biological composites. Deformation, crystallization and thermal, are modeled by nonlinear elasticity equations, phase-field models and heat equations, respectively

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(Chen et al., 2024). This category of strategies involves the use of multiscale techniques usually involving atomistic simulation with a continuum model in order to connect quantum mechanical interactions with behavior across macroscale scales of material performance. The combination helps to design more advanced materials that have the right characteristics to implement in aerospace, medicine, and energy systems.

Other new interdisciplinary areas are neuroscience, social systems and machine learning, with PDEs increasingly being used to describe dynamic behavior of networks, learning processes and emergent behavior. As an example, the work of the brain activity patterns in neural field equations are applied, and there are transport-related models based on the PDE which are applied to such things as migration or epidemic propagation. In addition, the combination of machine learning and PDE modeling has led to avenues of data-driven discovery where neural networks are used to approximate solution operators, or predict underlying dynamics using empirical data (Raissi et al., 2023). With the increasing accessibility of computation and subsequent data sensitivity of mathematical modeling, the domain of PDEs in interdisciplinary science is even more expanding, as it provides insights that have given wonders to what were thought to be unsolvable problems.



Problem Statement

Though tremendous progress has been achieved in modeling real life phenomena using partial differential equations, one of the most pressing challenges remains on defining interactions at multiple spatial and time scales in complex systems

Online ISSN 3007-3197
Print ISSN 3007-3189

accurately. Most models find it difficult to capture microscale processes without jeopardizing the feasibility of the model computationally or explainability at the macroscale. Besides, an important chasm occurs in interdisciplinary cooperation, in which theoretical advances in applied mathematics have not been maximized in modeling frameworks in particular fields. All these obstacles restrict the ability to develop strong predictive models of important problems in biomedicine, environmental science, and engineered systems.

Significant of Study

The study proposes the interdisciplinary role of applied mathematics and the PDE-based modeling technique in understanding the increasing interest in precise and scalable simulations in complicated systems. Through an overview of various case studies and computation frameworks, the paper develops links between mathematical and theoretical progress in the scope of mathematics and empirical and experimental problems in areas of natural and engineered settings. The derived knowledge can guide policy, the process of industrial production, clinical decisions and enlighten us on the complex nature of phenomena in various fields.

Aim of Study

This study take a closer look at the study of interdisciplinary applications of applied mathematics and partial differential equations in modeling such complex multi-scale phenomena and interrogate their integration into systems of biological, environmental, material, and computational systems. By way of compilation of state-of-art research and modeling strategies the paper intends to pinpoint best practice, methodological inhibitors, and future of multi-scale mathematical modeling.

Method

In this research, the qualitative systematic review methodology was applied in determining the interdisciplinary uses of applied mathematics and partial differential equations (PDEs) in the modeling of the complex multi-scale phenomena. The search was independent with references to electronic databases which are Scopus, Web of Science, ScienceDirect and IEEE Xplore. To include the most recent advances in both theoretical and practical fields, the journal articles focused on the topic within the context of peer review were included in the action regardless of their publication date and published in their journals during the period, January

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2021-July 2025. Boolean combinations of keywords were applied in the search strategy and these were partial differential equations, multi-scale modeling, interdisciplinary applications, applied mathematics, complex systems, and computational simulation. The studies had to meet the following inclusion criteria: the application of PDEs to the explicit modeling of real-world phenomena and the interdisciplinary integration in at least two fields (e.g. biology and mathematics or engineering and physics). Articles that contained insufficient clarity of methodological rigor, which merely contained pure mathematical derivation without application, or which preceded the year 2021 were not used as sources of analysis.

The total number of identified articles amounted to 168, and 97 of them were excluded according to the title and abstract screening, and 41 were excluded by the full-text review. A total of thirty studies with high quality were chosen and synthesized through the synthesis of themes. These studies of choice have been classified according to the area of the application; bio-medical sciences, environmental modelling, material and engineering sciences as well as computational sciences. Elaborate information was gathered on the kinds of PDEs employed (e.g., Navier-Stokes, reaction-diffusion, advection-diffusion), model scope (micro, meso, macro), modeling purpose (prediction, simulation, optimization) and interdisciplinary integration (tactics involved in linking across the scales or domains). They were coded thematically and facilitated by NVivo 14 program, which allowed to identify the general strategies of modeling, issues related to integrating operations on different scales, and the potential solutions in the computational domain (Thomas & Harden, 2008; Nowell et al., 2017).

The Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) protocol was adhered to in the process of conducting the review (Page et al., 2021). Triangulation was accomplished by enlisting the services of three independent reviewers to perform the calculation of screening and coding processes; and the inter-rater reliability assessed using Cohen s kappa with the result being 0.89, which means high concurrence. Critical Appraisal Skills Program (CASP) checklist was modified in accordance with computational modeling research to perform quality appraisal of the identified studies. The evaluation of methodological rigor was conducted regarding transparency, reproducibility and robustness of the

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PDE formulations employed. This broad look guaranteed a synthesized result to mirror accurately the present situation and advancing interdisciplinary settlement of PDE-structured problem tackling complicated surface and interior occasions.

Results

Table 1: ***Distribution of Reviewed Studies by Disciplinary Domain (n = 30)***

Disciplinary Domain	Number of Studies	Percentage (%)
Biomedical Sciences	9	30%
Environmental and Geophysical Modeling	7	23.3%
Materials Science and Engineering	6	20%
Computational and Data Sciences	5	16.7%
Social and Economic Systems	3	10%

Distribution of studies point out that the most represented sector (30%) in application of PDEs to model of complex system is biomedical sciences. This hegemony is indicative of the growing dependence on mathematical models to model biological phenomena like diffusion of drugs, formation of morphology in the tissues and growth of tumors. The high rate of popularity of environmental modeling (23.3%) and materials science (20) illustrates more on how PDEs contribute toward predictive modeling on natural and man-made structures. The fields are associated with demanding spatial-temporal modeling answers in real-to-life issues in climate dynamics, ground water movement, and structural deformation.

The other relatively smaller disciplines were the computational sciences (16.7%) and the socio-economic systems (10%). The problem view in which they appear is indicative of the increasing applicability of PDEs not just in the domain of physics. Specifically, the emergence of PDEs in social systems and finance suggests an emerging trend in basic human-ocentric fields to include in their arsenals decision support tools with a mathematical foundation. The evidence points at the overall pattern of an increasing interdisciplinary activity; in fact, the data are dominated by the growth of biology, environmental science, and engineering.

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Table 2: ***Types of Partial Differential Equations Utilized Across Domains***

Type of PDE	Number of Studies	Domains of Application
Reaction-Diffusion	10	Biomedical, Materials, Social Systems
Navier-Stokes	7	Environmental, Engineering, Computational Fluid Dynamics
Advection-Diffusion	5	Environmental, Biomedical
Phase-Field Models	4	Materials Science
Cable Equation	2	Neuroscience
Stochastic PDEs	4	Computational Modeling, Finance
Multi-scale Coupled PDEs	6	Mixed Domains (Biomedical, Environmental, Materials)

The most widely used PDEs in review studies were reaction-diffusion equations because in many cases they have been used to model biological pattern formation, chemical reactions, and spatiotemporal tissue and ecological dynamics. Navier-Stokes equations were next, being basic in fluid dynamics and environmental modelling. The fact that they are applicable in the description of complex flows, whether in engineered systems or natural systems, indicates the flexibility behind such an equation type in characterizing flows irrespective of whether they are streams of air and water or whether they include blood or oil.

Advection-diffusion and phase-field formulations were also prominent in terms of use in applications involving transport processes and phase transitions of materials. Also less common, Stochastic PDEs and multi-scale coupled systems, denote a sophisticated degree of modeling. Particularly, modeling domains that involve quantification of uncertainties or cross-scale interactions. The multiplicity of types of PDEs reveals interdisciplinary flexibility of mathematical tools to meet a wide range of disciplinary requirements, both in deterministic and probabilistic types of models.

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Table 3: ***Scale Representation in PDE Models (n = 30)***

Scale Level	Number of Studies	Example Applications
Microscale (cellular, molecular)	8	Tumor cell migration, nanoparticle transport
Mesoscale (tissue, region, process)	11	Groundwater transport, heart tissue modeling
Macroscale (organ/system/global)	7	Climate modeling, structural deformation
Multi-scale (coupled)	4	Cardiovascular flow + drug delivery

The scale analysis showed that there is an overlap in the studies at the mesoscale level (11 studies) containing tissues and regional transport modeling and subsystem interactions. This implies that most inter-disciplinary uses need a medium balance between computational effectiveness and biological or material authenticity. There was also a good representation of microscale models (8 studies), where cellular or molecular resolution is critical to local studies, such as signal transduction or drug-cell interactions in the case of biomedical research.

Also, there was under-representation of multi-scale and macroscale modeling which was of critical importance. Multi-scale models are inherently complicated and are computationally expensive, needing advanced numerical approaches and the scale-bridging hypotheses. The reduced macroscale models could be the indicator of preference of fine-grained simulation in new scientific fields of application. Multi-scale approaches are however getting more popular and this is an indication of the future, of models more and more connecting molecular to system behaviors in a single PDE framework.

Table 4: ***Purposes of PDE Modeling in Reviewed Studies***

Modeling Purpose	Number of Studies	Sample Keywords Used
Simulation Prediction	&12	Forecasting, scenario analysis

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Optimization	5	Design improvement, control systems
Pattern Formation	4	Biological morphogenesis, fracture modeling
Parameter Estimation	3	Inverse problems, data fitting
Decision Support	2	Policy analysis, clinical tools
Hybrid AI Integration	4	Physics-informed neural networks, PINNs

PDE models were mostly used in simulation and prediction (12 studies) and it indicated the importance of applied mathematics in predicting real life phenomenon under different conditions. The application is of particular interest in fields such as in meteorology, physiology and epidemiology, where it is important to predict the behavior of the system in circumstances of uncertainty. Even optimization problems were less common but of a very specific kind, as common in engineering and material science applications of designing superior systems or obtaining optimal results within limits.

The role of various useful functionalities PDEs serve is represented by the occurrence of the pattern formation module, parameter estimate and the decision support. Recently, the new phenomenon of hybrid artificial intelligence integration has been introduced in which neural networks using physics-informed neural networks (PINNs) play a key role. This hybridization is of particular value to ill-posed or data-limited problems and supports the cross disciplinary trend of models which are sufficiently rigorous mathematically, but computationally tractable.

Table 5: *Identified Challenges and Modeling Strategies in Multi-Scale PDE Applications*

Challenge	**Number of Studies**	**Modeling/Computational Strategies Used**
Scale Bridging and Coupling	14	Asymptotic methods, homogenization, nested solvers
Computational Cost	12	Sparse grids, adaptive mesh refinement, parallelization
Data Scarcity at Fine	7	Bayesian inference, surrogate models

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Scale		
Model Validation	6	Cross-disciplinary calibration, experimental mapping
Uncertainty Quantification	5	Stochastic PDEs, Monte Carlo simulations
Interdisciplinary Integration	8	Domain coupling, collaborative simulation platforms

The (scaling) problem of bridging scales was cited above as a major challenge in general, especially on taking a biological and environmental systems approach to processes that occur in a non-linear, interacting manner at micro and macro levels. To overcome this difficulty, effective strategies like asymptotic homogenization and nested solvers had become quite common so that one could reduce the model complexity avoiding a loss of key dynamics. The other common impediments that have been encountered include the computation cost where researchers have employed adaptive mesh refinement, parallel computing and sparse grid to attain optimized computational cost without necessarily jeopardizing the accuracy.

The lack of data and model verification was also an area of worry as especially in the neuro science and geophysical fields there is not enough high-resolution data. Employed Bayesian inference based on stochastic PDEs to deal with uncertainty and cross-disciplinary tools of validation technique were employed to ascertain models remained are based on empiricism. The increasing incorporation of artificial intelligence and machine learning tools suggests a dynamic initiative to attempt these difficulties with hybrid proposals of computation (along with physical theory and data inspired inductive probing).

Discussion

The partial differential equations (PDEs) have been increasingly adopted into interdisciplinary studies, where more complicated phenomena cross not only spatial scales but time scales that must be mathematically robust in their modeling. This review identified that the use of PDE-based methods has been the most popular in the biomedical sciences, and mainly in the simulation of physiological phenomena, including drug diffusion, tumor growth and electrophysiology. The dominance of reaction-diffusion models in this area can be explained by the fact that such systems

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have been described in the literature as an efficient approach to modeling spatial-temporal interactions in tissues (Geris et al., 2021). They are very easy to adapt and have the ability to incorporate with experimental data hence making them perfect representations of the dynamic processes in biology.

PDEs such as the Navier-Stokes equation and advection-diffusion equation were also seen to have been extensively integrated in environmental and geophysical modeling. They are fundamental to the computational modeling of fluid dynamics and the dispersion of pollutants and as such, their use in contemporary models of the climate has been growing sharply in demand to be coupled with thermodynamic and chemical reaction systems (Stevens et al., 2023). The liberty to represent atmospheric behavior over various scales allows predicting the weather patterns, the risk of disaster occurrence, and climate change. Scale-dependant complexity of such models however requires computational overhauls in the form of multigrid solvers and adaptive refinement strategies to remain computationally tractable.

Phase-field models, nonlinear elasticity equations were usually applied to material science problems, where they were used to capture stress, fracture and phase transformations in either composite or nanostructured materials. The applications celebrate the significance of PDEs in comprehending the interaction between microstructure and macro-behavior in advanced materials (Chen et al., 2024). Energy, biomedical and aerospace applications model such designs to be used in items whose performance and durability requires interactions at multiple scales. The joint methodology of the multi-scale coupling between atomistic models and continuum mechanics has seemed to provide a solution to enhance accuracy in each of the methods of application.

Interestingly, the study also noted that more research is applied to PDE in computational and data-driven sciences. Machine learning: Stochastic PDEs and physics-informed neural networks (PINNs) provide hybrid tools that are able to take advantage of machine learning, without sacrificing physical consistency (Raissi et al., 2023). In cases where there is little data and/or the data is noisy, these models are very useful in that they facilitate learning with very few instances of observation and they also facilitate the incorporation of domain knowledge. The usage of PDEs in such a setting is an indication of how modeling has changed in the sense that now

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Print ISSN 3007-3189

there are computationally intelligent systems to model processes in such residues as they are applicable across disciplines.

Although it has been overcoming the problems of scale-bridging, data and computational cost continues to be the challenge to some fields like neuroscience and social-economic modeling. Numerous studies cited that it was challenging to solve the problem of integrating models across micro, as well as macro, scales, with behaviors at various scales interacting non-linearly. Some of the advanced approaches like asymptotic homogenization and adaptive mesh refinement have assisted in dealing with this problem (Oden et al., 2022), but these strategies are usually multicomponent and constrained to general deployment. Equally, absence of quality data in validating the models has been an obstacle especially in the biological systems and climate prediction at a long term time cycle.

Nonetheless, the review confirms that one can reach a conclusion that PDEs play a crucial role as the means of representing and analyzing multi-scale phenomenon. They are also modifiable to fit in any interdisciplinary problem as they are flexible mathematically, and compatible with numerical methods. Notably, this practice of applying PDEs was once theoretical and now it is directly applicable in decision support, making of policies, and execution of simulations in real-time. This shift can already be seen as a sign of not only developments in applied mathematics but also the realization that integrated modeling methodologies were becoming increasingly necessary even in the approach to modern scientific and societal problems.

Future Direction

To expand interoperability with real-time data acquisition systems in adaptive and personalized modeling applications such as precision medicine and smart environmental monitoring, future work should tackle how to make PDE models interoperate with these data systems. More research into combining machine learning and PDE-based modelling should also be done focusing on interpretability and scalability. Open-source multi-scale modeling platforms which can be multi-disciplinary, user-friendly and computationally efficient will thus enable a wider interdisciplinary collaboration as well as the utilization of the models across underrepresented areas like economics and behavioral sciences.

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Limitations

One of the limitations of this work was that it was restricted by the used sources of published peer-reviewed studies which may possibly leave out the use of preprints or subject-specific reports which are not listed in the chosen databases. Moreover, the qualitative review itself might have added subjectivity into the subject of the classification of modeling methods and challenges. The critique was also so concentrated on high-reputation journals, that it may not completely reflect the richness of new cross-disciplinarity work, particularly of the developing world or inaccessible centers of academia.

Conclusion

Partial differential equations have been and still are the basis of interdisciplinary modeling, and their use along with the unprecedented modularity allows to simulate complex systems in many different fields. The studies examined show that PDE-based models become increasingly mixed with the data science methods, enlarging their scope and forecast capabilities. PDEs form the basis of understanding systems and systems too complex to be governed empirically through observation and analysis, in the case of biomedical engineering, in weather forecasting, and in climate modeling. The ability of PDEs to enable a solution to practical problems will only increases and become more fundamental and broad-reaching with the changing nature of computational resources and algorithmic frameworks.

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