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Numerical Solution and Analysis of Oldroyd-B Fluid Flow in the Presence of Magnetic Field

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ABSTRACT

The numerical solution of the Oldroyd-B fluid model plays a crucial role in understanding and predicting complex fluid flow behaviors. In this paper, we present a comprehensive investigation of the numerical solution, truncation error for the convergence of the scheme, and Von Neumann stability analysis for the finite difference scheme of Oldroyd-B fluid model. We considered a one-dimensional Oldroyd-B flow equation in a channel, incorporating the effects of slip parameter and magnetic field. A forward-in-time and central-in-space finite difference method is used for the discritization of the corresponding one dimensional partial differential equation. We evaluate the convergence and stability of the numerical scheme by analyzing the truncation error and performing Von Neumann stability analysis. A MATLAB code is made to implement the numerical solution and demonstrate the convergence and stability through graphical representations. We also test an example and checked the effects of Reynolds number, slip parameter and magnetic field on the flow problem. Our all, the results indicate the reliability and effectiveness of the employed numerical scheme. The presented analysis showcases the potential of the numerical solution and provides a foundation for future studies involving multi-term variables and nonlinear partial differential equations.

Keywords: Oldroyd-B Fluid ,Finite Difference Method, Von Neumann Stability Analysis.

Introduction

Fluid mechanics is an exciting and essential field of engineering that studies the behavior of fluids at rest or in motion. It well used to understand the flow of liquids and gases to determine how they interact with objects and surfaces. Fluid mechanics well be used in a wide range of engineering fields, including aeronautics, energy, and transportation. It well used to analyze the effects of air flow over airplane wings and to design cooling systems for cars. It is also used to

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understand the behavior of rivers, oceans, and other bodies of water, and to design systems that can take advantage of the flow of water. Fluid mechanics can also be used to study the behavior of complex fluids, such as blood and other biological fluids. This can help us design better medical devices and treatments. Overall, fluid mechanics is an incredibly useful and important field of study, and engineers are constantly finding new ways to use it to improve our lives and the world around us.

Overall, the investigation of non-Newtonian fluid flow behavior becomes more difficult than the Newtonian fluid. Hartnett and Kostic have mentioned that[1] predictions about heat transfer in non-Newtonian fluids under laminar flow regime yield inaccurate estimates. Non-Newtonian fluid flow in the container becomes one of the crucial issues in fluid dynamics. Initially, Stokes[2] has studied non-Newtonian fluid motion in this regard. Then Rajagopal et al.[3] have derived exact results on the motion of classical linearly viscous fluid in oscillatory flow. Rajagopal and Bhatnagar[4] have made the discussion about the mathematical modeling of non-Newtonian fluids flow. Moreover, Mahmood et al.[5] have examined Oldroyd-B fluid model for the longitudinal and torsional oscillation of very long dowel through Bessel function solutions. In their work, Hayat et al.[6] obtained exact expressions for velocity and shear stress in terms of sine wave for the flow of a second grade fluid using Laplace and finite Hankel transforms. They found that there exist five specific solutions for the flow of an Oldroyd-B fluid, which include, Stokes and modified Stokes problem, periodic Poiseuille flow, non-periodic flows and symmetric flow.

Non-Newtonian fluids have found significance because of the various applications in engineering, indus-tries and biology. Non-Newtonian fluids can be found in many materials and products like polymeric fluids, gels, blood, foam, detergents, fruit pastes, macromolecular fluids and many more practical uses. Among many non-Newtonian fluids models, the Jeffrey's fluid model is one of the widely used models [7, 8]. Due to its comparatively easy mathematical representation, Jeffrey's fluid model gives a good mathematical description about the properties of the non-Newtonian fluids. It includes the relaxation and retardation phenomenon and is able to model the behavior of viscoelastic materials. The governing equations of Jef-frey's fluid have also been studied on the basis of suspended fluid. The generalize Oldroyd-B fluid is special distinct subclass of Jeffrey's fluids that has been discovered to resemble the reaction to several diluted polymeric liquids.References[9, 10, 11, 12] include some current research on the generalize Oldroyd-B fluids. Sutton has outlined the essential electromagnetic connections[13]. Zheng at[13] addressed the magneto-hydrodynamic (MHD) flow bounded by two plates with slip boundary conditions furthermore obtained every precise solution in terms of the Fox Hfunction using certain transform methods. The MHD flow of a genreralised Oldroyd-B fluid in a spheroid pipe along with a porous region, corrospondly, were calculated by Khan at[14, 15].

Due to its employment in many fields of science, engineering as well as technology, notably in the substantial rectification as well as chemical along with nuclear industries, geophysics furthermore bio-engineering, every flow of non-

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Newtonian fluids has lately attained enormous relevance. The Oil reservoir engineering is also very interested in this research of non-Newtonian fluid flow. Non-Newtonian fluids are categorized based on how they behave under shear for a number of reasons. A fluid is said to be Newtonian if every shear stress along with shear rate have a linear relationship that results in a constant viscosity. As there exist fluids related a second order fluid, a converted Maxwell fluid along with an Oldroyd-B fluid A and B a certain unquestionably non-Newtonian although nevertheless exhibit a stable viscosity, stable viscosity association is not invariably a Newtonian fluid association. Every classical Newtonian fluid as well as Maxwell fluid are two particular examples of the Oldroyd-B fluid, which has recently gained a distinctive standing among the several fluids of the rate type[16, 17].

Mathematical formulation

An important part of the equation is that the below an incompressible Oldroyd-B fluid is confined between two finite rigid plates under the influence of an applied magnetic field is applied under every supposition about a little magnetic Reynddes number, See Fetecay at [20].
Investigated the one dimensional case:

$$\frac{\partial U}{\partial t} = \frac{v\partial^2 U(x,t)}{\partial x^2} - \frac{\mu}{\rho} v\varphi U(x,t) \kappa \sigma\beta^2 U(x,t) + f(x,t). \quad (1)$$

where $v = \frac{\mu}{\rho}$ is the kinematic viscosity of the fluid, ρ is the density of the fluid and μ is the dynamic viscosity coefficient of the fluid, where U(x,t) represents the velocity field. Khan et al.[21] investigated a generalized Oldroyd-B fluid model in a porous medium by considering the effects of Hall current.

$$\frac{\partial U}{\partial t} = \frac{v\partial^2 U(x,t)}{\partial x^2} - \frac{\varphi}{(1-\iota\psi)} v\varphi U(x,t) \kappa \sigma\beta^2 U(x,t) + f(x,t). \quad (2)$$

where κ represents the permeability coefficient of the porous medium, φ denotes its porosity, ψ is the Hall parameter, β^2 indicates the magnetic field intensity, and σ represents the electrical conductivity.

Initial and Boundary Conditions

The flow moves between vertical strips along the x-axis and there is a constant flow on top and a non-constant flow on bottom, and at $t = 0$ the flow is at rest.

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$$U(x,0) = u_0 Sin(\pi x) \qquad 0 \le x \le 1$$

$U(0,t) = 0 \;\; U(1,t) = 0 \;\; t > 0$

As a result of the above research in this fluid, I am stimulated. Here is one dimensional Oldroyed-B fluid with Hall current influence.

$$\frac{\partial U}{\partial t} = \frac{\nu \partial^2 U(x,t)}{\partial x^2} - \frac{\nu\varphi U(x,t)\,\kappa}{\sigma\beta^2 U(x,t)} + f(x,t). \qquad (3)$$
$$(1-\iota\psi)$$

Non-dimensionalization

Simplifying the governing partial differential equation (3) can be achieved by non-dimensionalizing them. This process involves identifying the key variables that describe the flow problem and replacing them with dimensionless variables. For this, h is the characteristic length and U_0 is the velocity. Then, the variables in the equation are transformed as follows:

$$t^* = \frac{\nu t}{h^2}, \; x^* = \frac{x}{h}, \; U^* = \frac{U}{U_0},$$

In a nondimensional format, Equation (3) can be expressed as follows (omitting the asterisks):

$$\frac{\partial U}{\partial t} = \frac{1}{Re}\frac{\partial^2 U(x,t)}{\partial x^2} - \psi U(x,t) - M_0 U(x,t) + f(x,t) \qquad (4)$$

Here Re is Reynolds number, ψ is slip parameter and M_0 magnetic parameter.

Numerical Technique

In this section, we will drive finite difference scheme such as Forward in Time and Central in Space (FTCS) method to discretize PDE (4). The discretization is given below:

Discretization Method

Let's a choose an integers $n > 0$, $k > 0$ and a discretization of time variable $0 < t \le T$, such as $t^k = k\Delta t$ where $k = 0, 1, 2, 3...T$ and a discretization of space variable $a \;\; x_i \;\; b$ is $x_i = a + ih$ where $i = 0, 1, 2, 3...n$. Using forward in time and central in space method to our (3),

$$\frac{U_i^{k+1} - U_i^k}{\Delta t} = \frac{1}{Re}\frac{U_{i+1}^k - 2U_i^k + U_{i-1}^k}{h^2} - \psi U_i^k(x,t) - M_o U_i^k(x,t)$$

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$$|(x,t) + f(x,t) \qquad (5)$$

$$\Delta t U^{k} \; {}^{k} - 2U_{1} + U \qquad {}^{k} \qquad {}^{k}$$

$$U^{k+1} - U^{k}|(x,t) =$$

$$_{i+1} \quad _{i}$$

$$|_{(x,t)} - \Delta t\psi U \; |_{(x,t)} - M_0 \Delta t U \; |_{(x,t)} + f(x,t) \qquad (6)$$

$$_{i} \qquad _{i} \qquad \frac{}{Reh^2} \qquad _{i} \qquad _{i}$$

$$^{k} \quad U \quad ^{k} - 2U_{i-1} + U \qquad ^{k} \qquad ^{k}$$

$$_{i} \quad U^{k+1}|(x,t) = U^{k}|(x,t) + \Delta t \qquad _{i} \qquad _{i}$$

$$_{i+1} \quad _{i}$$

$$Reh$$

$$|_{(x,t)} - \Delta t\psi U \; |_{(x,t)} - M_0 \Delta t U \; |_{(x,t)} + f(x,t) \qquad (7)$$

Convergence of the numerical scheme

In numerical analysis, a numerical scheme is said to be convergent if the solution obtained from the discretized problem approaches the exact solution of the original differential equation as the mesh size approaches zero. For linear initial and boundary value problems, the Lax equivalence theorem [22] establishes that, for a well-posed linear problem, a finite difference approximation is convergent if and only if it is both consistent and stable. Therefore, consistency alone does not guarantee the reliability of a numerical method; stability is also essential to ensure that the computed solution converges to the true solution [23].

Definition:

The finite difference method (FDM) is consistent wherever

$$\lim_{h \to 0} T(x,t) = \lim_{d \to 0} (P_h U(x,t) - PU(x,t)) = 0 \qquad (8)$$

Let's $P = \frac{d}{dt}$ stand for differential operator on U in a linear differential equation and P_h be a corresponding finite difference operator.

PU = f stand for $U''(x,t) = f(x,t)$ if $P\left(\frac{d}{dx}, \frac{d^2}{dx^2}\right) =$

Thus, the local trunction error is define as

$$T(x,t) = P_h U - PU$$

Note that local trunction errors depend on the solution in the finite difference stencil but not on the solution globally.

$$\partial U \quad v\partial^2 U(x,t)$$

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$$\frac{\partial t}{} = \frac{}{\partial x^2}$$

$$\frac{v\varphi}{\kappa} \frac{}{U(x,t)} \frac{\sigma\beta^2}{\rho(1-i\psi)} U(x,t) + f(x,t). \quad (9)$$

At a gird point x_i, we can approximate $U(x_i, t^k)$ using nearby function values to get a finite difference for the 2nd order derivative with some error in the approximation.

In the finite difference method (FDM), we replace the differential equation at each grid point x_i,where the T^k is the Local truncation error as

$$T_i^k = \frac{1}{Re}\frac{\partial^2 U(x_i,t^k)}{\partial x^2} - \psi U(x_i,t^k) - M_0 U(x_i,t^k) + f(x_i,t^k). \quad (10)$$

By using Taylor expansion at point x_i, we have

$$U(x_{i+1},t^k) = U + hU_x + \frac{h^2U_{xx}}{2!} + \frac{h^3U_{xxx}}{3!} + \frac{h^4U_{xxxx}}{4!} + \frac{h^5U_{xxxxx}}{5!} + \ldots \quad (11)$$

$$U(x_{i-1},t^k) = U - hU_x + \frac{h^2U_{xx}}{2!} - \frac{h^3U_{xxx}}{3!} + \frac{h^4U_{xxxx}}{4!} - \frac{h^5U_{xxxxx}}{5!} + \ldots \quad (12)$$

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$$U(x_i, t^{k+1}) = U + \frac{hU_t}{1!} + \frac{h^2U_{tt}}{2!} + \frac{h^3U_{ttt}}{3!} + \frac{h^4U_{tttt}}{4!} + \frac{h^5U_{ttttt}}{5!} + \ldots \quad (13)$$

Using equations (11) to (13), the equation (10) will become

$$T_i^k = \frac{\Delta t U_{tt}^k}{12} - \frac{h^2}{Re}\left[\frac{h^4U_{xxxx}^k}{12} + \ldots\right]\Big|_{(x_i,t^k)} - \psi U(x_i, t^k) - M_0U(x_i, t^k) + f(x_i, t^k) - \left[U_t + \frac{\Delta t}{2!} + \ldots\right]\Big|_{(x_i,t^k)} \quad (14)$$

$$\Delta t T_i^k = \left[U_{xx}^k + \frac{\Delta t}{12} U_{xxxx}^k + \ldots\right]\Big|_{(x_i,t^k)} - \frac{\psi h^2}{Re} - \psi U(x_i, t^k) - M_0U(x_i, t^k) + f(x_i, t^k) - \left[U_t + \frac{\Delta t}{2!} U_{tt}^k + \ldots\right]\Big|_{(x_i,t^k)} \quad (15)$$

From(10) We have

$$\Delta t^k \frac{h^2}{4} \frac{\Delta t}{2!} = \frac{h^2}{Re} U_{xxxx}\Big|_{(x_i,t^k)} - U_{tt}\Big|_{(x_i,t^k)} + (h^4, \Delta t^2). \quad (16)$$

The von Neumann stability analysis

A method for evaluating the stability of finite difference schemes used with linear partial differential equations is the von-Neumann stability analysis (also known as the Fourier stability analysis)[28]. After be-ing briefly described in a 1947 publication by British scholars Crank and Nicolson[29], the approach, which is based on the Fourier decomposition of numerical error, was created at Los

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Alamos National Laboratory. In a piece[30] that John von Neumann co-authored, this method was treated more strictly. The von Neu-mann algorithm relies on the breakdown of mistakes into Fourier series. The one-dimensional incompressible Oldroyd-B fluid is a good example of the process (1)
The von Neumann algorithm relies on the breakdown of mistakes into Fourier series. To demonstrate the process, examine the one-dimensional incompressible Oldroyd-B fluid (1),

$$\frac{\partial U}{\partial t} = \frac{1}{Re}\frac{\partial^2 U(x,t)}{\partial x^2} - \psi U(x,t) - M_0 U(x,t) \qquad (17)$$

The discritization of the above equation using forward in time and central in space method, we have

$$U^{k+1} = \lambda U^k_{i+1} + \lambda U^k_{i-1} + (1 - 2\lambda - \Delta t\psi - \Delta t M_0)U^k_i, \qquad (18)$$

where $\lambda = \frac{\Delta t}{Re}$ is As, Fourier transform we have

$$U_i^k = \frac{1}{\sqrt{2\pi}} \int_{-\frac{\pi}{h}}^{\frac{\pi}{h}} e^{i\xi(ih)} \hat{U}^k d\xi$$

$$\upsilon^k_{i+1} = \frac{1}{\sqrt{2\pi}} \int_{-\frac{\pi}{h}}^{\frac{\pi}{h}} e^{i\xi(i+1)h} \hat{U}^k d\xi$$

$$\upsilon^k_{i-1} = \frac{1}{\sqrt{2\pi}} \int_{-\frac{\pi}{h}}^{\frac{\pi}{h}} e^{i\xi(i-1)h} \hat{U}^k d\xi$$

putting above results into (18), we gets

$$\hat{U}^{k+1} \quad \lambda \quad \pi \; h$$

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$$U = \sqrt{\quad} e^{i\xi(i+1)h}\hat{U}k + \sqrt{\quad} e^{i\xi(i-1)h}\hat{U}k + (1-2\lambda-\Delta t\psi-\Delta tM_0)e^{i\xi(ih)}\hat{U}k$$

$d(\xi)$ (19)

i 2π $-\pi$
(ξ)
2π $-\pi$
(ξ)
2π $-\pi$
(ξ)

Taking common on R.H.S, we gets

$$U^{k+1} = \sqrt{\quad} \quad (\xi)$$

2π $-\pi$

$$[\lambda e^{i\xi h} + \lambda e^{-i\xi h} + (1-2\lambda-\Delta t\psi-\Delta tM_0)]\hat{U}^k$$

$d(\xi)$ (20)

The discrete fourier transform define has

$$U^{k+1} = \frac{1}{\sqrt{\quad}} \quad e^{i\xi(ih)}\hat{U}^{(k+1)}d(\xi)(21)$$

i 2π $-\pi$

comparing(20) and(21),we get

$$\sqrt{\quad} \quad \sqrt{\quad} \quad e^{i\xi(ih)}\hat{U}^{k+1}d\xi =$$

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$$\int_{-\pi}^{\pi} e^{i\xi(ih)}[\lambda e^{-i\xi(ih)}e^{i\xi h}U + \lambda e^{-i\xi h} + (1-2\lambda-C_1-C_2)]\hat{U}^k d\xi \quad (22)$$

$$k+1(\xi) = [(1-2\lambda-\Delta t\psi-\Delta tM_0)+\lambda(e^{i\xi h}+e^{-i\xi h})]U^k \quad (\xi)$$

(23)The discrete fourier transform is unique, which implies

$$U^{k+1}_{(\xi)} = [(1-2\lambda-\Delta t\psi-\Delta tM_0)+\lambda(e^{i\xi h}_{(\xi)}+e^{-i\xi h})]\hat{U}^k$$

$$= g(\xi)\hat{U}^k \quad (\xi)$$

(24)

where $g(\xi) = (1-2\lambda-\Delta t\psi-\Delta tM_0)+\lambda(e^{i\xi h}+e^{-i\xi h})$ as known as growth factor

Assuming that $|g_{(\xi)}| \le 1$, then $|U^{k+1}| \le |\hat{U^k}|$,

The finite difference scheme is stable; now let's look at $|g_{(\xi)}|$,

Immediately we have

(ξ)

(ξ)

$$g_{(\xi)} = (1-2\lambda-\Delta t\psi-\Delta tM_0)+\lambda(e^{i\xi h}+e^{-i\xi h})$$

$$g_{(\xi)} = (1-2\lambda-\Delta t\psi-\Delta tM_0)+2\lambda Cos(\xi h)$$

$$g_{(\xi)} = 1-\Delta t\psi-\Delta tM_0-2\lambda(1-Cos(\xi h))$$

$$g(\xi) = 1-\Delta t\psi-\Delta tM_0 - 4\lambda Sin^2\left(\frac{\xi h}{2}\right)$$

(25)

although we extremity to know $|g_{(\xi)}| \le 1$ either $-1 \le g_{(\xi)} \le 1$

$$0 \le Sin^2\left(\frac{\xi h}{2}\right) \le 1$$

$$-4\lambda \le -4\lambda Sin^2\left(\frac{\xi h}{2}\right)$$

$$1-4\lambda \le 1-4\lambda Sin^2\left(\frac{\xi h}{2}\right)$$

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$$-\Delta t\psi - \Delta t M_0 + 1 - 4\lambda \le -\Delta t\psi - \Delta t M_0 + 1 = 4\lambda Sin\left(\frac{\ }{2}\right)$$

$$-1 \le 1 - 4\lambda \le g_{(\xi)} \le 1 \qquad (26)$$

extremely on taking $-1 \le 1 - 4\lambda \le 1$ we can agreement that $g(\xi) \le 1$ and that mention the stability, Conse-quently, a necessary condition for the stability of the finite difference method (FDM) is

$$-1 \le 1 - 4\lambda \le 1, \quad \frac{\ }{2} \ge 4\lambda, \quad \lambda \le \frac{1}{2}$$

Although we cannot predict what would happen if this condition is broken, it does establish an upper bound for the stability. In the below figures, it is shown graphically that, if we take $\lambda = 2\Delta t/h^2 = 0.5$ then the scheme is stable as shown in (1(b)) and (2(b)). While in graphs, (1(a)) and (2(a)), we took $\lambda = 2\Delta t/h^2 > 0.5$, the graphs diverges.

Figure of U when 2dt/(h.h) = 1.5

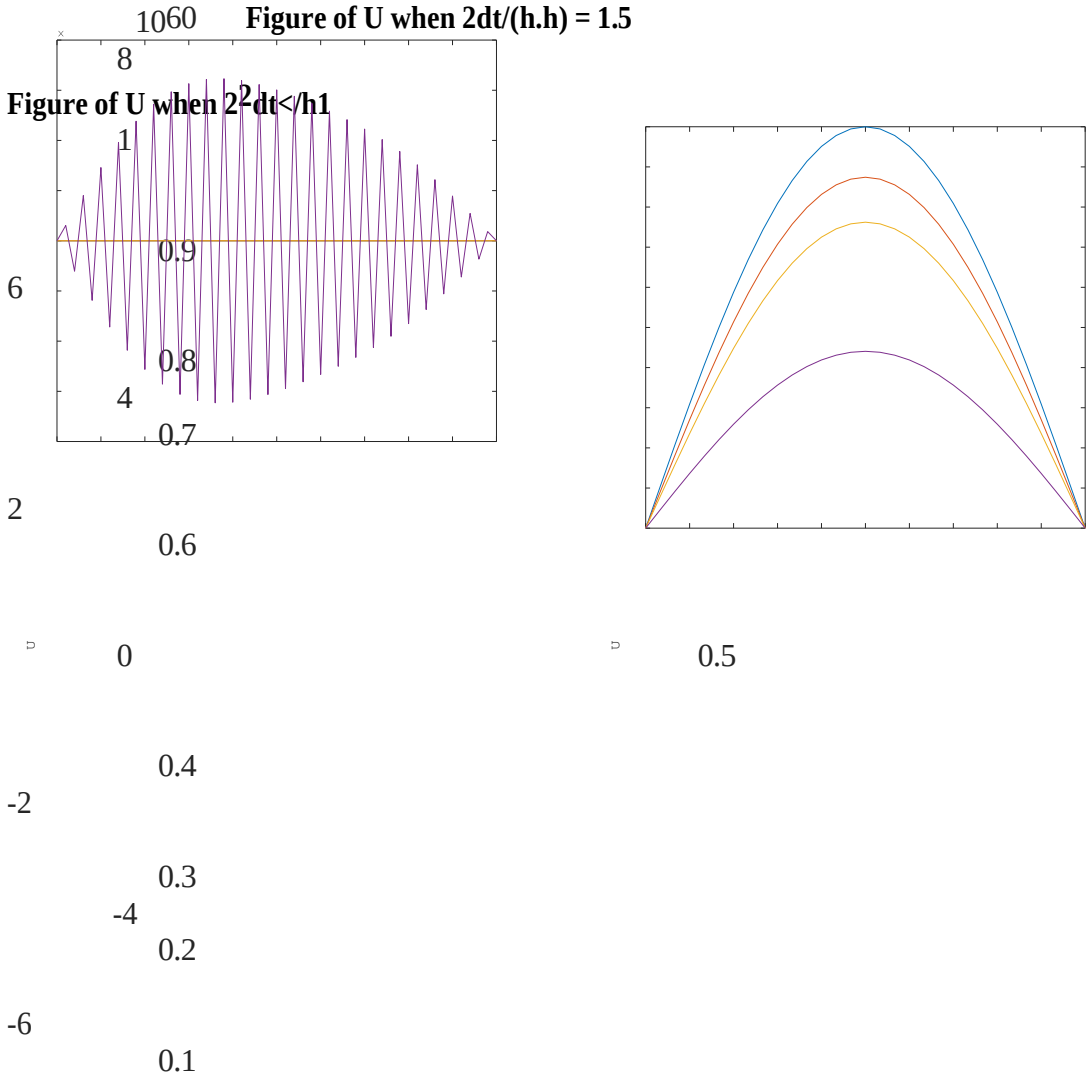

Figure of U when 2^2dt</h1

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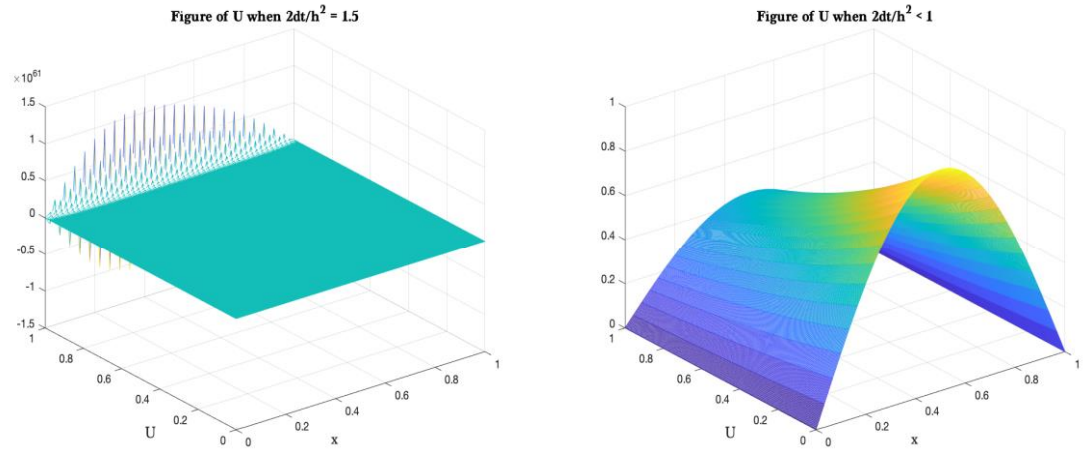
-8
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.91
x
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
x

(a) Initial and consecutive approximations (b) Initial and consecutive approximations

Figure 1



(a) Initial and consecutive approximations (b) Initial and consecutive approximations

Figure 2

Table

Where error $|E = U_{Ex}| U_{App}$, here we took U_{Ex}... is exact solution, m is space step size, n is time step size, from table (1). It is shown the error estimate is good for our numerical scheme. In (1), when we take $m = 8$ and $n = 20$, then we get first order O(h) error.

Similarly, by increasing space step size and time step size the error decreased and we got error up to order two $O(h^2)$ at $m = 40$, $n = 60$ and order three $O(h^3)$ at $m = 60$, $n = 80$ up to so on.

1.1. Numerical results and discussion

The numerical calculations for the space-time derivative parameter, the relaxation time parameter λ, the Reynolds number Re, the slip parameter ψ, the magnetic parameter M_o For convenience, the non-dimensional time t is chosen as 1.2. In figure (3(a)) show numerical and exact solution graphs on $m = 8$

Table 1: Comparison of the maximum errors at time

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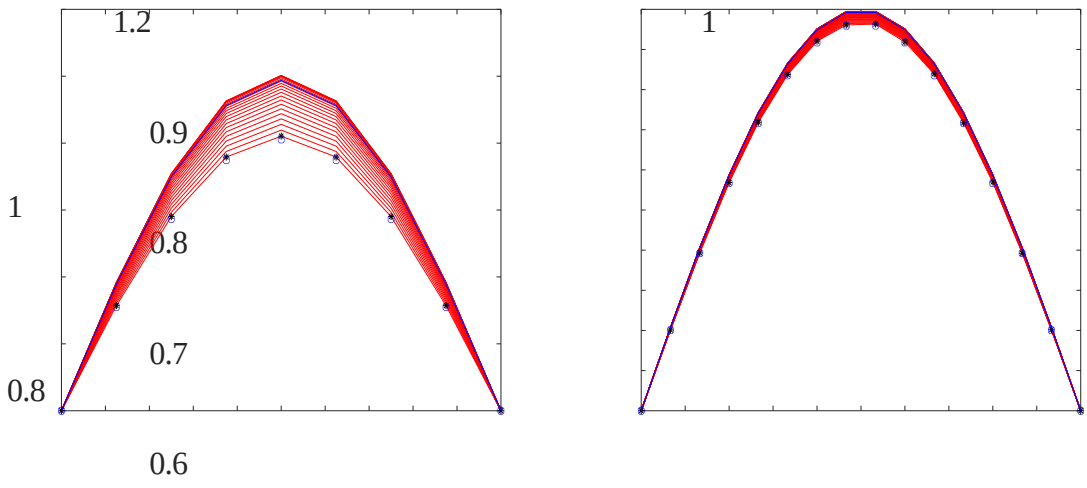
S.no	*m*	*n*	Re	E
1	08	20	-2	0.9975×10^{2}
2	15	30	-2	0.3950×10^{2}
3	25	40	-04	3.0801×10^{10}
4	30	50	-04	1.9044×10^{10}
5	40	60	-05	7.6132×10^{10}
6	50	70	-05	3.7389×10^{10}
7	60	80	-05	2.0955×10^{10}
8	80	100	-06	8.4483×10^{10}

and $n = 20$. Here blue doted lines shows exact solution defined in (1) section and red lines shows numerical results at different m. similarly figure (3(b)) shows numerical results at $m = 15$ and $n = 30$, and in (4(a)) $m = 40$ and $n = 60$. The numerical and exact solutions are same in graph (4(b)), where we took $m = 60$ and $n = 80$. This proves that our numerical scheme converges and error minimizes when we increase m and n values.



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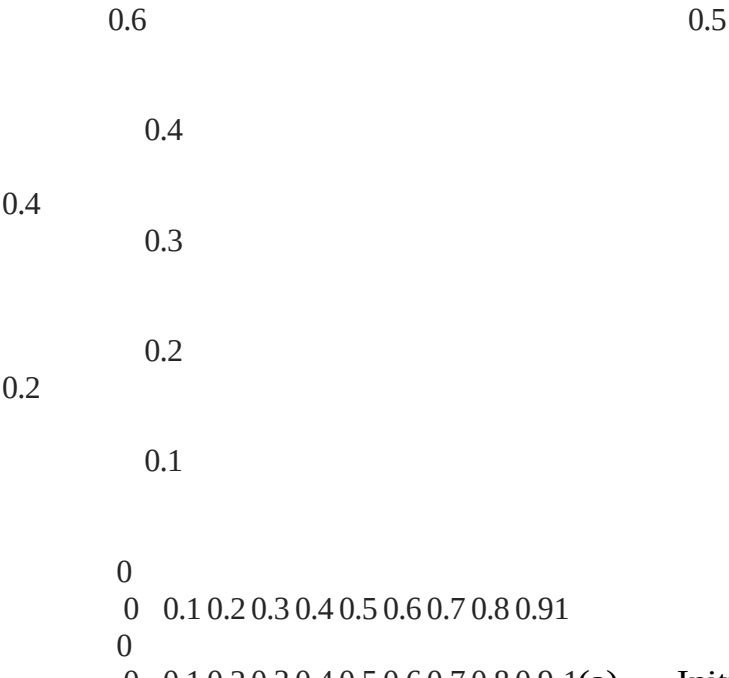
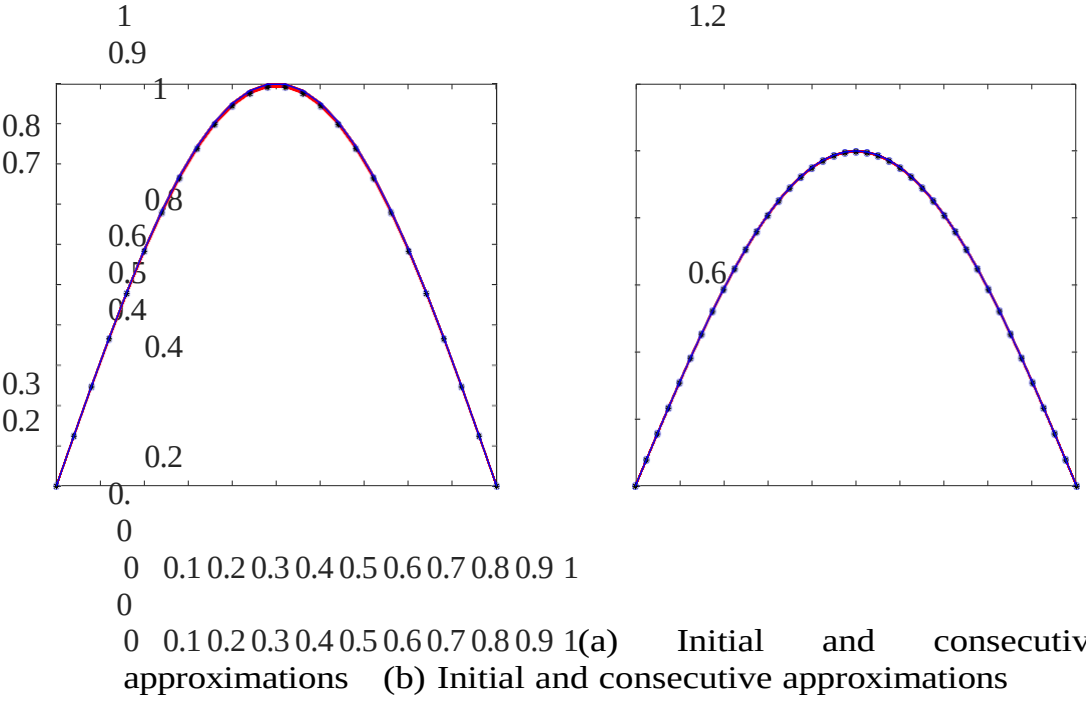


(a) Initial and consecutive approximations (b) Initial and consecutive approximations

Figure 3



(a) Initial and consecutive approximations (b) Initial and consecutive approximations

Figure 4

Example: In this example, we consider $f(x) = \frac{\pi J_0 M_0}{} \exp(\frac{-\pi x}{})$ **as a**

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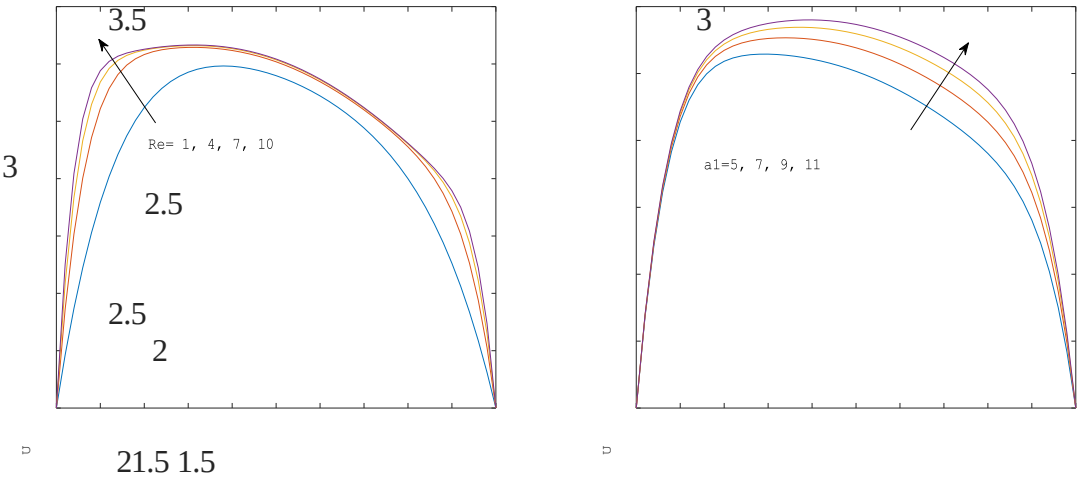
source term.

8ρ a_1

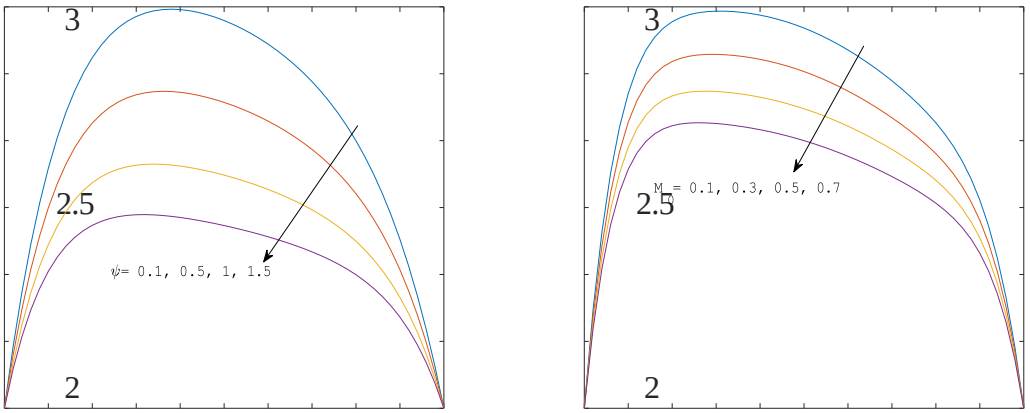
here J_o is current density, M_o is the magnetization of the magnet and a_1 with of the magnet and electrods

and the graphical results are shown below.



(a) Re (b) a1

Figure 5



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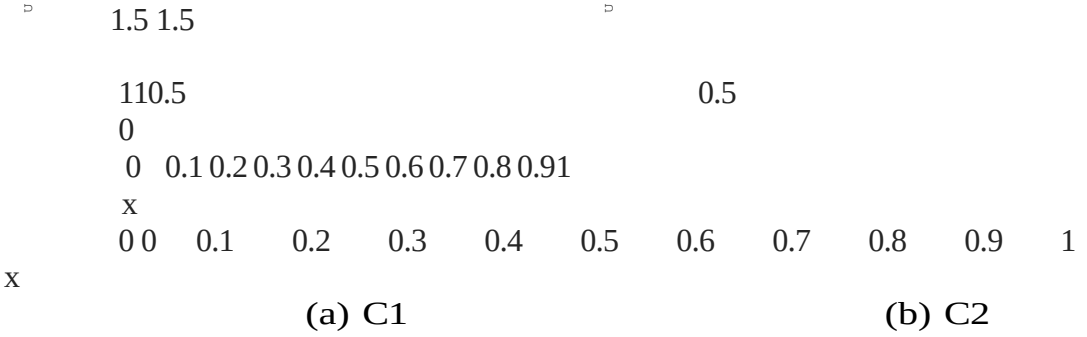

(a) C1 (b) C2

Figure 6

The effects of Reynolds number are shown in figure (5(a)), The Reynolds number is the proportion of inertial drive and thick constraint that decides flow behavior whether it is turbulent or laminar. The velocity boundary layer increases when we increase values of Re from 1 to 10. An increment in boundary layer flow alludes to a circumstance where there's an increment within the flow rate or speed of the fluid inside the boundary layer. The boundary layer is the lean layer of fluid that shapes adjoining to a strong surface in a streaming fluid. It plays a pivotal part in deciding the in general flow behavior and characteristics. When the boundary layer flow increases, it ordinarily comes about in changes within the stream properties and marvels. he effects of the magnetization of the magnet $a1$ shown in figure (5(b)). As shown in graph, flow of the graph increases quickly when we increase magnetictization parameter values $a1$. A slip parameter ψ and magnetic parameter M_0 effects are shown in (6(a)) and (6(b)). Fluid velocity decreases when we increase small increment in ψ and magnetic parameter M_0. A small increase in slip parameter decrease fluid flow quickly and similarly a small increment in magnetic parameter also decrease flow velocity quickly.

Conclusion

In this paper, we studied the behavior of a one-dimensional Oldroyd-B fluid that flows in a channel by consolidating slip parameter and magnetic field effects. Our study involves the utilization of numerical methods to obtain a solution and assess the convergence of the 1D partial differential equation. Specifically in finite difference methods, we employ a forward-in-time and central-in-space method to solve the 1D partial differential equation and evaluate the convergence and stability analysis of the finite difference scheme by considering truncation error and Von Neumann stability analysis. To demonstrate the reliability and effectiveness of our approach, we present a numerical example and showcase the convergence and stability through graphical representations using a MATLAB code. Our findings indicate that the employed numerical scheme, known as FTCS, exhibits robustness and can be extended to handle other multi-term variables and nonlinear partial differential equations in both space and time domains or parabolic equations. In conclusion, our study

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highlights the potential of the FTCS numerical scheme for analyzing complex fluid dynamics and can be applied to various scenarios involving space and time variables.

References

J. P. Hartnett and M. Kostic, "Heat transfer to Newtonian and non-Newtonian fluids in rectangular ducts," in Advances in Heat Transfer, vol. 19, pp. 247–356, Elsevier, 1989.

G. G. Stokes, "On the effect of the rotation of cylinders and spheres about their axis in increasing the logarithmic decrement of the arc of vibration," 1886.

K. R. Rajagopal, "Longitudinal and torsional oscillations of a rod in a non-Newtonian fluid," Acta Mechanica, vol. 49, no. 3, pp. 281–285, 1983.

K. R. Rajagopal and R. K. Bhatnagar, "Exact solutions for some simple flows of an oldroyd-b fluid," Acta Mechanica, vol. 113, no. 1, pp. 233–239, 1995.

A. Mahmood, N. A. Khan, I. Siddique, and S. Nazir, "A note on sinusoidal motion of a viscoelastic non-Newtonian fluid," Archive of Applied.

T. Hayat, A. M. Siddiqui, and S. Asghar, "Some simple flows of an oldroyd-b fluid," International Journal of Engineering Science, vol. 39, no. 2, pp. 135–147, 2001.

G. B. Jeffrey, The motion of ellipsoidal particales immersed in a viscous fluid. Proc. Royal Society of London. Series A, Containing Paper of a mathematical Physical Character, Volume 120, Issue 715, pp. 161-179, 102(161), 1922.]

Kahshan, M, Lu, D.; Siddiqui, A.M. A Jeffrey fluid model for a porous-walled channel: Application to flat plate dialyzer. Sci. Rep. 2019, 9, 15879.

L.C. Zheng, Y.Q. Liu, X.X. Zhang, Slip effects on MHD flow of a generalized Oldroyd-B fluid with fractional derivative, Nonlinear Analysis: Real World Applications, 13, 513-523 (2012).

H.T. Qi, M. Y. Xu, Some unsteady unidirectional flows of a generalized Oldroyd-B fluid with fractional derivative.

J. Zhao, L. Zheng, Z. Zhang, F. Liu, Unsteady natural convection boundary layer heat transfer of fractional Maxwell viscoelastic fluid over a vertical plate, International Journal of Heat and Mass Transfer, 97, 760-766 (2016).

J. Zhao, L. Zheng, X.X. Zhang, F. Liu, X. Chen, Unsteady natural convection heat Transfer past a vertical flat plate embedded in a porous medium saturated with Fractional Qldroyd-B fluid, Journal of Heat Transfer, 139, 012501 (2017).

G.M. Sutton, A. Sherman, Engineering magneto-hydrodynamics, McGraw-Hill, New York (1965).

Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about
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Online ISSN 3007-3197
Print ISSN 3007-3189
http://amresearchreview.com/index.php/Journal/about

Constantin Fetecaua, Corina Fetecau, M. Kamran, D. Vieru, Exact solution for the flow of a generalized Oldroyed-B fluid induced by a constantly accelerating plate between Side walls perpendicular to the plate, J.Non-Newtonian Fluid Mech. 156, 189201 (2009).

Kahshan, M.; Lu, D.; Siddiqui, A.M. A Jeffrey fluid model for a porous-walled channel Application to flat plate dialyzer. Sci. Rep. 2019, 9, 15879. [CrossRef]. [3] M.Khan, K. Maqbool, T.Hayat, Influence of Hall current on the flows of a generalized Oldroyed-B fluid in a porous space, Acta Mechanica, 184, 1-13(2006).

T.HayataA.M.SiddiquibS.Asghara, Some simple flows of an Oldroyd-B fluid.

MahmoudAyyad, Stabilized variational formulation of an Oldroyd-B fluid flow Equations on a Graphic Processing Unit (GPU) architecture.

Joel H. Ferziger. Milovan, PeriC, Computarional Methods for Fluid Dynamics,3., rev.ed. - Berlin; Heidelberg; New York; Barcelona; Hong Kong; London; Milan; Paris;Tokyo: Springer, 2002.

Rachel Levy . Michael Shearer ,Partial Differential Equations: An Introduction to Theory and Applications, Princeton March 1, (2015).

Constantin Fetecaua, Corina Fetecau, M. Kamran, D. Vieru, Exact solution for the flow of a generalized Oldroyed-B fluid induced by a constantly accelerating plate between side walls perpendicular to the plate, J.Non-Newtonian Fluid Mech. 156, 189201 (2009).

M.Khan, K. Maqbool, T.Hayat, Influence of Hall current on the flows of a generalzied Oldroyed-B fluid in a porous space, Acta Mechanica, 184, 1-13(2006).

Richtmyer and Morton, 1967.

G. B. Jeffrey. The motion of ellipsoidal particles immersed in a viscous fluid. Proc. Roy. SOCA., 102(161), (1922).

G. B. Jeffrey, The motion of ellipsoidal particales immersed in a viscous fluid. Proc. Royal Society of London. Series A, Containing Paper of a mathematical Physical Character, Volume 120, Issue 715, pp. 161-179, 102(161), 1922.

Y.Q. Liu, L.C. Zheng, X.X. Zhang, Unsteady MHD Couette flow of a generalized Oldroyd-B fluid with fractional derivative.

[11] L.C. Zheng, Y.Q. Liu, X.X. Zhang, Slip effects on MHD flow of a generalized Oldroyd-B fluid with fractional derivative, Nonlinear Analysis: Real World Applications, 13, 513-523 (2012).

[5] H.T. Qi, M. Y. Xu, Some unsteady unidirectional flows of a generalized Oldroyd-B fluid with fractional derivative.

J. Zhao, L. Zheng, Z. Zhang, F. Liu, Unsteady natural convection boundary layer heat transfer of fractional Maxwell viscoelastic fluid over a vertical plate, International Journal of Heat and Mass Transfer, 97, 760-766 (2016).

J. Zhao, L. Zheng, X.X. Zhang, F. Liu, X. Chen, Unsteady natural convection heat transfer past a vertical flat plate embedded in a

Online ISSN 3007-3197
Print ISSN 3007-3189

porous medium saturated with fractional Qldroyd-B fluid, Journal of Heat Transfer, 139, 012501 (2017).

Y. Jiang, H. Qi, H. Xu, X. Jiang, Transient electroosmotic slip flow of fractional OldroydB fluids, Microfluid Nanofluid, 21, 1-10 (2017).

G.M. Sutton, A. Sherman, Engineering magnetohydrodynamics, McGraw-Hill, New York (1965).

Joel H. Ferziger. Milovan, PeriC, Computarional Methods for Fluid Dynamics,3., rev. ed. - Berlin; Heidelberg; New York; Barcelona; Hong Kong; London; Milan; Paris; Tokyo: Springer, 2002.

Rachel Levy . Michael Shearer ,Partial Differential Equations: An Introduction to Theory and Applications, Princeton March 1, (2015).

E. Isaacson, H. B. Keller. Analysis of Numerical Methods.

Crank, J.; Nicolson, P. (1947), ”A Practical Method for Numerical Evaluation of Solutions of Partial Differential Equations of Heat Conduction Type”, Proc. Camb. Phil. Soc., 43: 50–67, doi:10.1007/BF02127704.

Charney, J. G.; Fjørtoft, R.; von Neumann, J. (1950), ”Numerical Integration of the Barotropic Vorticity Equation”, Tellus, 2: 237–254, doi:10.3402/tellusa.v2i4.8607.