

The Dynamics of Correlated Shocks: Option Pricing and Estimation in a Regime-Switching Co-Jump Model

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ABSTRACT

This paper proposes and estimates a regime-switching stochastic volatility co-jump (RS-SVCJ) model in which the parameters governing the joint distribution of simultaneous jumps in returns and volatility are allowed to vary across macroeconomic regimes governed by a continuous-time finite-state Markov chain. Within each regime, asset prices and stochastic variance follow a generalization of the Eraker–Johannes–Polson (2003) model, while the co-jump correlation, jump intensity, and jump size distribution are all regime-dependent. We derive a semi-closed-form expression for the conditional characteristic function of the log-asset price by solving a system of coupled Riccati-type ordinary differential equations, from which European option prices are computed via the fast Fourier transform. A Bayesian MCMC estimator is developed that jointly samples model parameters, latent volatility, jump times, jump sizes, and the latent regime sequence via a combination of Gibbs sampling and forward-filtering backward-sampling. Simulation experiments confirm excellent finite-sample performance. Applied to twenty years of daily S&P 500 returns, the model identifies two well-defined regimes a tranquil bull market and a crisis bear market with markedly different co-jump correlations and jump intensities. Bayes factors strongly favour the full RS-SVCJ

specification over all nested alternatives, and the model produces substantially smaller implied volatility pricing errors than single-regime benchmarks. Our results confirm that the correlation between return jumps and volatility jumps is strongly state-dependent, with important consequences for tail risk measurement and derivative pricing.

Keywords: regime-switching; co-jumps; stochastic volatility; option pricing; Bayesian MCMC; characteristic function; implied volatility

1. Introduction

The accurate pricing of financial derivatives and the reliable estimation of asset price dynamics have remained among the most consequential challenges in quantitative finance since the publication of the seminal Black–Scholes–Merton framework in the early 1970s. Despite the mathematical elegance and widespread adoption of that model, decades of empirical evidence have consistently demonstrated its inability to capture several pervasive stylised facts of financial return distributions, including leptokurtosis, return skewness, volatility clustering, and the well-documented phenomenon of the implied volatility smile. The classical assumption of constant volatility and normally distributed returns enshrined in the Black–Scholes model is systematically violated in virtually every liquid options market, where out-of-the-money puts and deep in-the-money calls command implied volatilities substantially different from those of at-the-money contracts [1, 2]. The cumulative weight of this empirical evidence has motivated a rich and expanding literature devoted to the development of more flexible and empirically realistic models of asset price dynamics. A major strand of this literature has focused on enriching the continuous diffusion of the Black–Scholes framework by superimposing discontinuous jump components onto the asset price process. The seminal contribution of Merton [3] introduced a compound Poisson jump process to model sudden large changes in asset prices, such as those triggered by macroeconomic announcements, geopolitical shocks, or corporate events. This jump-diffusion paradigm was subsequently extended and refined in numerous directions. Kou [4] proposed a double-exponential jump-diffusion model with asymmetric jump sizes that generates closed-form option prices and naturally reproduces the leptokurtic return distribution and the volatility smile

observed in equity markets. Kou's framework highlighted that downside jumps tend to be larger in magnitude than upside jumps a stylised asymmetry with far-reaching implications for risk management. Duffie, Pan, and Singleton [5] further advanced this programme by developing the general class of affine jump-diffusion models, for which characteristic functions can be obtained in closed form, facilitating efficient option pricing via Fourier transform methods. The broad empirical relevance of jumps was confirmed by Andersen, Benzoni, and Lund [6], who found strong evidence for jumps in S&P 500 returns using high-frequency data, and by Barndorff-Nielsen and Shephard [7], whose non-parametric bipower variation approach enabled the disentanglement of the continuous and jump components of quadratic variation.

A parallel and complementary literature recognised that jumps alone are insufficient to explain the full term structure of implied volatility and the persistence of volatility shocks. This motivated the development of stochastic volatility models, most notably the affine square-root model of Heston [8], and subsequently models combining stochastic volatility with jumps in returns. Bates [9] unified these two strands by allowing for stochastic volatility and Poisson jumps in the return process, while retaining the tractability afforded by the affine structure. The most comprehensive model in this class the stochastic volatility with co-jumps (SVCJ) model was introduced and empirically investigated by Eraker, Johannes, and Polson [10], who found overwhelming evidence for simultaneous jumps in both returns and volatility using S&P 500 and Nasdaq 100 index data. Their likelihood-based Bayesian MCMC estimation strategy represented a methodological landmark, enabling the joint estimation of parameters, latent volatility paths, and jump times and sizes from daily return data alone. The SVCJ model demonstrated that ignoring jumps in volatility beyond jumps in returns leads to substantially inferior model fit, particularly during periods of market stress such as the 1987 crash and the crises of 1997 and 1998 [10, 11].

While the SVCJ framework and its relatives provide a rich description of return dynamics, they are nonetheless premised on parameter constancy over time an assumption that is difficult to reconcile with the empirically observed structural

shifts in financial markets during different macroeconomic regimes. Market behaviour during periods of economic expansion differs fundamentally from behaviour during recessions, credit crises, or episodes of extreme uncertainty. Hamilton's [12] regime-switching framework offered an elegant solution: the parameters governing the drift and volatility of asset returns are allowed to switch among a finite number of states, governed by an unobservable continuous-time Markov chain. When applied to financial markets, this regime-switching structure naturally accommodates bull and bear regimes, high- and low-volatility states, and the clustering of extreme events all of which have been extensively documented empirically [13, 14]. Regime-switching models have been shown to generate richer implied volatility surfaces than constant-parameter models and to improve substantially the pricing of options with medium- to long-dated maturities, where the risk of regime transition is economically material. Recent work by Shirzadi et al. [15] on American option pricing under regime-switching jump-diffusion models with meshfree methods, and by Chen [16] on fast and accurate computation of regime-switching jump-diffusion option prices using Laplace transforms, has revived interest in numerical methods tailored to these combined frameworks.

A separate but increasingly integrated stream of research has focused on the co-jump structure of multivariate asset price processes that is, the phenomenon whereby two or more assets experience simultaneous discontinuous price changes driven by common macroeconomic or systemic shocks. Co-jumps are of critical importance for portfolio risk management, derivatives pricing on multiple underlyings, and the identification of systemic risk. Barndorff-Nielsen and Shephard [7] developed early non-parametric procedures for detecting co-jumps using high-frequency data, while Jacod and Todorov [17] provided rigorous asymptotic theory for co-jump tests in the presence of microstructure noise. Subsequent contributions by Bibinger and Winkelmann [18] and Caporin et al. [19] refined these econometric tools considerably. The importance of co-jumps has been confirmed across equity, bond, and foreign exchange markets [20, 21], and their dynamic properties including increased co-jump intensity during crisis episodes such as the COVID-19 pandemic have been documented by multiple recent studies [22, 23].

Within the parametric modelling literature, spread option pricing under regime-switching jump-diffusion models with idiosyncratic and systematic shocks was explored by Caldana, Fusai, and colleagues [24], who demonstrated that explicitly modelling systematic shocks those that simultaneously affect multiple underlying assets provides a materially more accurate representation of the joint distributional dynamics than models relying solely on correlated Brownian motions.

Despite the substantial advances described above, there remain important gaps in the existing literature that this paper seeks to address. First, most existing models that incorporate both regime-switching and jumps treat the jump component as either purely idiosyncratic or as driven by a single common shock, without allowing for correlated but distinct jump shocks across the return and volatility processes whose joint structure is itself regime-dependent. The degree of correlation between return jumps and volatility jumps a key determinant of the skewness of the risk-neutral return distribution and hence of the shape of the implied volatility smile is rarely allowed to vary with the macroeconomic regime, despite strong theoretical and empirical reasons to expect it to do so. During crisis regimes, for example, large negative return jumps are systematically accompanied by large positive volatility jumps, generating an asymmetry that is absent or attenuated in tranquil regimes [25]. Second, the Markov-switching threshold stochastic volatility framework recently proposed by Benbachir et al. [26] and the forward-starting option pricing models of [27] demonstrate that the interaction between regime-switching and volatility dynamics generates qualitatively distinct implied volatility patterns that cannot be replicated by single-regime models; yet neither framework incorporates co-jumps. Third, while Bayesian MCMC estimation has proven powerful for the SVCJ model, its extension to regime-switching settings with correlated co-jumps raises non-trivial inferential challenges associated with the intractability of the likelihood function, the high dimensionality of the latent state space, and the identification of regimes from data alone.

This paper makes several interrelated contributions to this literature. We propose a novel regime-switching co-jump model in which the joint dynamics of asset returns and stochastic volatility including simultaneous jumps in both

processes are governed by a continuous-time finite-state Markov chain. Within each regime, the return and volatility processes follow a generalized version of the SVCJ model, but the parameters of the co-jump distribution, including the mean, variance, and correlation of jump sizes, are regime-dependent. This framework nests several existing models as special cases, including the SVCJ model of Eraker et al. [10], the regime-switching stochastic volatility model without jumps, and the bivariate regime-switching jump-diffusion model of [24]. The regime-switching structure allows the model to capture the empirically documented state-dependence of jump risk a feature that is particularly relevant for pricing options with longer maturities, for which the probability of regime transitions is non-negligible.

On the option pricing side, we derive the moment generating function of the log-asset price under each regime using the Feynman–Kac theorem for regime-switching diffusions with jumps, and show that it satisfies a system of coupled ordinary differential equations that admits a semi-closed form solution under the affine parametric restrictions standard in the stochastic volatility literature. From this characteristic function, option prices can be computed efficiently using the fast Fourier transform method of Carr and Madan [28] or the more recent COS method of Fang and Oosterlee [29]. The regime-switching structure introduces additional state variables specifically the current regime probabilities into the pricing kernel, generating a richer and more flexible implied volatility surface that can simultaneously reproduce the level, slope, and curvature of market-observed smiles across maturities and strikes. Recent advances in numerical methods for regime-switching option pricing, including the implicit-explicit compact difference scheme of Chen and Li [30] and the meshfree finite-point method of Shirzadi et al. [15], provide efficient computational tools that complement our semi-analytical pricing results.

On the estimation side, we develop a Bayesian MCMC algorithm that jointly samples the model parameters, the latent volatility path, the jump arrival times and sizes, and the sequence of regimes from the posterior distribution. Our algorithm builds on the data-augmentation approach of Eraker, Johannes, and Polson [10] and the regime-inference methodology of Kim, Nelson, and Startz [31], combining

Gibbs sampling for the continuous latent states with a forward-filtering backward-sampling algorithm for the discrete regime sequence. We demonstrate via simulation that the algorithm recovers the true model parameters with high accuracy and that the precision of regime identification improves with sample size in a manner consistent with asymptotic theory. An empirical application to S&P 500 index returns provides strong evidence in favor of the full regime-switching co-jump specification relative to nested alternatives, with Bayes factors decisively preferring models that allow for regime-dependent co-jump correlation. The estimated regime sequences align naturally with well-known episodes of market stress, lending economic credibility to the identified regimes. The remainder of the paper is organized as follows. Section 2 introduces the regime-switching co-jump model, provides formal definitions of the component processes, and discusses the economic motivations for the key modelling choices. Section 3 derives the moment-generating function of the log-asset price under the risk-neutral measure and establishes its semi-closed form, from which the option pricing formulae are obtained. Section 4 presents the Bayesian MCMC estimation methodology, including details of the prior specification, sampling algorithm, and convergence diagnostics. Section 5 reports simulation evidence on the finite-sample performance of the estimator. Section 6 presents the empirical application to S&P 500 index returns, including parameter estimates, regime identification, option pricing performance, and model comparison results. Section 7 concludes with a discussion of extensions and directions for future research. Technical proofs are collected in the Appendix.

2. The Regime-Switching Co-Jump Model

2.1 Markov Chain and Regime Structure

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space satisfying the usual conditions. We model the macroeconomic regime by a continuous-time, time-homogeneous Markov chain $\{Z(t), t \geq 0\}$ taking values in the finite state space $\mathcal{K} = \{1, 2, \dots, K\}$, with generator matrix $Q = (q_{ij})$, where $q_{ij} \geq 0$ for $i \neq j$ and $q_{ii} = -\sum_{j \neq i} q_{ij}$. We focus primarily on the two-regime case $K = 2$, labelled the *tranquil* regime ($Z = 1$) and the *crisis* regime ($Z = 2$), but all results extend to $K > 2$. The regime process $Z(t)$ is assumed to be independent of the Brownian motions and Poisson processes that

drive the state variables conditional on the parameter set, but it drives the parameters of those processes, thereby inducing state-dependence in all distributional features of asset returns.

2.2 Asset Price and Variance Dynamics

Let $S(t)$ denote the price of the risky asset and $V(t)$ the instantaneous variance. Under the physical (objective) measure $\mathbb{P}$, the dynamics are governed by the following system of stochastic differential equations:

$$dS(t)/S(t) = \mu(Z(t)) dt + \sqrt{V(t)} dW_{-S}(t) + (e^{\{J_{-S}\}} - 1) dN(t),$$

$$dV(t) = \kappa(Z(t))[\theta(Z(t)) - V(t)] dt + \sigma_{-V}(Z(t)) \sqrt{V(t)} dW_{-V}(t) + J_{-V} dN(t),$$

where $W_{-S}(t)$ and $W_{-V}(t)$ are standard Brownian motions with instantaneous correlation $dW_{-S}(t) \cdot dW_{-V}(t) = \rho(Z(t)) dt$, capturing the leverage effect. The regime-dependent parameters $\mu(Z)$, $\kappa(Z)$, $\theta(Z)$, $\sigma_{-V}(Z)$, and $\rho(Z)$ represent, respectively, the drift, mean-reversion speed, long-run variance, variance volatility ('vol-of-vol'), and leverage correlation in state Z . The Poisson process $N(t)$ has regime-dependent intensity $\lambda(Z(t))$, and triggers simultaneous jumps in the log-return and variance a co-jump.

2.3 Co-Jump Distribution

The co-jump sizes (J_{-S}, J_{-V}) are drawn from a bivariate distribution that is also regime-dependent. Conditional on regime $Z = s$, we specify:

$$J_{-V} / Z=s \sim \text{Exponential}(\eta_{-V}(s)), \quad \text{so that } J_{-V} \geq 0 \text{ always,}$$

$$J_{-S} / J_{-V}, Z=s \sim \text{Normal}(\mu_{-J}(s) + \rho_{-J}(s) J_{-V}, \sigma_{-J}^2(s)),$$

where $\mu_{-J}(s)$ is the mean log-jump size, $\sigma_{-J}(s)$ the idiosyncratic jump standard deviation, $\rho_{-J}(s)$ the co-jump regression coefficient linking return jump size to the concurrent volatility jump, and $\eta_{-V}(s)$ the rate parameter of the exponential volatility jump distribution. The marginal distribution of J_{-S} is thus a Normal-Exponential mixture whose mean, variance, and skewness are all functions of the regime. The parameter $\rho_{-J}(s)$ is the key novelty of our framework: it governs the degree to which large volatility jumps are accompanied by large negative (or positive) return jumps, and we allow it to differ across regimes. Empirically, we expect $\rho_{-J}(2) < \rho_{-J}(1) < 0$, reflecting the empirical regularity that the return–

volatility co-jump correlation is most negative during crisis episodes [10, 25].

2.4 Special Cases

The RS-SVCJ model nests several important special cases: (i) setting $K = 1$ recovers the SVCJ model of Eraker et al. [10]; (ii) setting $\lambda(s) = 0$ for all s gives the regime-switching Heston model studied by He and Lin [32]; (iii) setting $\rho_{-}J(s) = 0$ for all s yields a regime-switching model with independent return and volatility jumps; (iv) imposing parameter homogeneity across regimes in (i)–(iii) recovers their constant-parameter analogues. Table 1 summarizes the nested model hierarchy.

Model	Regimes (K)	Jumps	Co-jump corr.
Black–Scholes	1	No	
Heston (1993)	1	No	
Merton (1976)	1	Return only	
SVCJ (Eraker et al. 2003)	1	Co-jump	Fixed
RS-Heston	K	No	
RS-SVJ	K	Return only	
RS-SVCJ (this paper)	K	Co-jump	Regime-dependent

Table 1. Nesting structure of the RS-SVCJ model. All special cases are obtained by parameter restrictions on the full model.

3. Option Pricing

3.1 Risk-Neutral Measure

To derive option prices, we transform the dynamics to the equivalent risk-neutral measure $\mathbb{Q}$. By Girsanov's theorem applied to the Brownian component and by specification of a compensator adjustment for the jump component, the risk-neutral dynamics of the log-price $X(t) = \log S(t)$ satisfy:

$$dX(t) = [r - \delta - \frac{1}{2}V(t) - \lambda(Z(t))\zeta(Z(t))]dt + \sqrt{V(t)}dW_{-}S^{\mathbb{Q}}(t) + J_{-}SdN^{\mathbb{Q}}(t),$$

$$dV(t) = \kappa^{\mathbb{Q}}(Z(t))[\theta^{\mathbb{Q}}(Z(t)) - V(t)]dt + \sigma_{-}V(Z(t))\sqrt{V(t)}dW_{-}V^{\mathbb{Q}}(t) + J_{-}V$$

$$dN^{\wedge @}(t),$$

where r is the risk-free rate, δ the dividend yield, and $\zeta(s) = E^{\wedge @}[\tilde{e}^{\wedge \{J_S\}} - 1 \mid Z = s]$ is the mean jump-induced return compensation, computed from the risk-neutral co-jump distribution. Under the risk-neutral measure the Brownian motions remain correlated with $\rho(Z(t))$, and the jump intensity becomes $\lambda^{\wedge @}(s)$. The risk premium for regime transitions is embedded in the generator $Q^{\wedge @}$, allowing the model-implied physical and risk-neutral regime probabilities to differ a feature empirically important for matching the level of the volatility risk premium [33].

3.2 Conditional Characteristic Function

The central pricing object is the conditional moment generating function (MGF) of the log-return, defined for complex argument u as:

$$\Psi(u, V, s, \tau) = E^{\wedge @}[\tilde{e}^{\wedge \{u X(T)\}} \mid X(t) = x, V(t) = V, Z(t) = s],$$

where $\tau = T - t$ is time to expiry. We seek an affine solution of the form:

$$\Psi(u, V, s, \tau) = \exp\{A_s(u, \tau) + B_s(u, \tau) V + u x\},$$

where the functions $A_s(u, \cdot)$ and $B_s(u, \cdot)$ satisfy a coupled system of Riccati-type ODEs. Applying the Feynman–Kac theorem extended to Markov-switching jump-diffusions, and using the affine structure of the model, we obtain the following system for $s = 1, \dots, K$:

$$\begin{aligned} \partial B_s / \partial \tau &= \frac{1}{2} u(u-1) - (\kappa^{\wedge @}(s) - \rho(s) \sigma_- V(s) u) B_s + \frac{1}{2} \sigma_-^2 V(s)^2 B_s^2 + \\ &\quad \lambda^{\wedge @}(s) [\Phi_- J(u, B_s, s) - 1], \end{aligned}$$

$$\partial A_s / \partial \tau = (r - \delta) u + \kappa^{\wedge @}(s) \theta^{\wedge @}(s) B_s + \sum_{\{j \neq s\}} q_{- \{sj\}}^{\wedge @} [A_{-j} - A_s],$$

with terminal conditions $A_s(u, 0) = 0$ and $B_s(u, 0) = 0$ for all s . Here $\Phi_- J(u, B, s)$ is the joint Laplace transform of the co-jump vector $(J_S, J_- V)$ under $@$ in regime s , which under our Normal-Exponential specification takes the closed form:

$$\Phi_- J(u, B, s) = \exp\{u \mu_- J(s) + \frac{1}{2} \sigma_-^2 J(s)^2 u^2\} \cdot \eta_- V(s) / (\eta_- V(s) - \rho_- J(s) u - B),$$

provided $\eta_- V(s) - \rho_- J(s) u - Re(B) > 0$. The coupling between regimes enters through the off-diagonal terms $q_{- \{sj\}}^{\wedge @} (A_{-j} - A_s)$ in the ODE for A_s . This system does not admit a closed-form solution in general but is solved efficiently and accurately by standard Runge–Kutta methods (e.g., RK45), with computation times of order milliseconds for typical parameter values and a two-regime specification.

3.3 European Option Prices

Given the characteristic function $\phi(u) = \Psi(iu, V, s, \tau)$ (i.e., the Fourier transform of the log-return density), European call option prices are obtained via the fast Fourier transform (FFT) method of Carr and Madan [28]:

$$C(K, \tau) = (e^{-\alpha \log K} / \pi) \int_0^\infty \text{Re}[e^{-iv \log K} \psi(v)] dv,$$

where $\psi(v) = e^{-\tau} \phi(v - (\alpha+1)i) / (\alpha^2 + \alpha - v^2 + i(2\alpha+1)v)$, with damping parameter $\alpha > 0$ chosen to ensure integrability. Alternatively, the Fourier-cosine (COS) method of Fang and Oosterlee [29] provides a complementary approach that is particularly accurate for short maturities. To account for the current (unknown) regime, option prices are computed as a weighted average across regimes:

$$C(K, \tau \mid V, \pi) = \sum_s \pi_s \cdot C_s(K, \tau \mid V),$$

where $\pi_s = P(Z(t) = s \mid \text{data})$ is the filtered regime probability at time t , obtained from the Bayesian MCMC algorithm described in Section 4. This regime-probability weighting naturally introduces an additional source of option pricing uncertainty that is absent from single-regime models [10]: investors not only face uncertainty about the current level of volatility, but also about the current macroeconomic regime.

3.4 Implied Volatility Surface Properties

The RS-SVCJ model generates a richer implied volatility surface than its constant-parameter counterparts. In the tranquil regime, the model produces a relatively flat volatility term structure and a mild negative skew, consistent with low jump intensity and small co-jump correlations. In the crisis regime, the model generates a steeply inverted volatility term structure and a pronounced negative skew, driven by high jump intensity, large negative co-jump correlation, and elevated variance-of-variance. The transition between regimes introduces additional volatility of the implied volatility surface a feature empirically documented by Gonçalves and Guidolin [34] and consistent with the time-varying dynamics of the S&P 500 options market. At intermediate maturities, the model captures the 'smile flattening' as time to expiry increases, since the probability of regime transition attenuates the asymmetry generated by crisis-state jump risk. This mechanism is qualitatively distinct from the mean-reversion of volatility in the Heston model and provides a

more realistic description of the observed maturity structure of implied volatility skews.

4. Bayesian MCMC Estimation

4.1 Data Augmentation and Posterior

Let $Y = \{y_{-1}, \dots, y_{-T}\}$ denote the observed daily log-returns where $y_{-t} = X(t) - X(t-1)$. The complete-data vector is augmented with the latent volatility path $V = \{V_{-1}, \dots, V_{-T}\}$, the jump arrival indicators $\xi = \{\xi_{-1}, \dots, \xi_{-T}\} \in \{0, 1\}^T$, the jump sizes $J = \{(J_{-S}^t, J_{-V}^t) : \xi_{-t} = 1\}$, and the discrete regime sequence $Z = \{Z_{-0}, Z_{-1}, \dots, Z_{-T}\}$. The posterior of interest is:

$$p(\Theta, V, \xi, J, Z \mid Y) \propto p(Y \mid \Theta, V, \xi, J, Z) \cdot p(V, \xi, J \mid \Theta, Z) \cdot p(Z \mid Q) \cdot p(\Theta),$$

where Θ collects all model parameters and $p(\Theta)$ is the prior. The key computational challenge is the intractability of the marginal likelihood when integrating over the latent states. Data augmentation resolves this by treating (V, ξ, J, Z) as additional unknowns and iterating a Gibbs sampler over the full conditional distributions [10, 48].

4.2 Prior Specifications

Priors are chosen to be weakly informative, centered on values consistent with the extant literature while placing positive mass over a wide range of plausible values. Specifically: $\kappa(s)$, $\theta(s)$ and $\sigma_{-V}(s)$ are assigned Gamma priors with means matching Eraker et al. [10] estimates; the leverage correlation $\rho(s) \sim \text{Uniform}(-1, 0)$; the jump intensity $\lambda(s) \sim \text{Gamma}(a_{-}\lambda, b_{-}\lambda)$ with $a_{-}\lambda = 1$, $b_{-}\lambda = 0.1$ (prior mean of one jump per ten days); mean log-jump size $\mu_{-J}(s) \sim \text{Normal}(-0.03, 0.01^2)$ (mild negative prior on mean jumps); idiosyncratic jump standard deviation $\sigma_{-J}(s) \sim \text{Half-Normal}(0.05^2)$; the key co-jump regression coefficient $\rho_{-J}(s) \sim \text{Normal}(-0.5, 0.5^2)$; volatility jump rate $\eta_{-V}(s) \sim \text{Gamma}(2, 10)$; and the off-diagonal generator elements $q_{-12}, q_{-21} \sim \text{Gamma}(1, 5)$ (prior mean sojourn time of 5 days). Risk premium parameters are assigned Normal priors with zero mean and standard deviation 0.5. We verify prior sensitivity in a robustness analysis reported in Section 5.

4.3 Gibbs Sampling Steps

The Gibbs sampler iterates over six blocks. Each block draws from the

corresponding full conditional distribution:

Block 1: Latent Volatility Path V

Given (Θ, ξ, J, Z, Y) , the log-volatility $\log V_t$ is approximately Gaussian by a log-normal approximation to the CIR process, enabling a Kalman filter-based smoothing step. We use the particle Gibbs with ancestor sampling (PGAS) algorithm of Lindsten, Jordan, and Schön [35] to draw the entire volatility path jointly, which avoids the slow mixing associated with single-move samplers and is particularly important in the presence of jumps.

Block 2: Jump Indicators ξ and Jump Sizes J

Each jump indicator ξ_t is drawn from its full conditional Bernoulli distribution, whose parameter involves the ratio of the complete-data likelihood under the jump and no-jump hypotheses. The jump size (J_{S^t}, J_{V^t}) conditional on $\xi_t = 1$ is drawn from a bivariate normal distribution (after the change of variables induced by the Normal-Exponential prior), updated by the observed return and volatility increment. This step is the analogue of the jump augmentation step in Eraker et al. [10], extended to regime-dependence.

Block 3: Regime Sequence Z

The discrete regime sequence $\{Z_t\}$ is sampled using the Forward-Filtering Backward-Sampling (FFBS) algorithm, which targets the exact joint posterior of the regime path. The forward pass computes the filtered regime probabilities $P(Z_t = s \mid Y_{1:t}, V_{1:t}, \xi_{1:t}, J_{1:t}, \Theta)$ recursively using the Chapman–Kolmogorov equations; the backward pass samples a realisation of the entire regime path by drawing $Z_T \mid \pi_T$ and then $Z_t \mid Z_{t+1}, \pi_t$ for $t = T-1, \dots, 1$. This multi-move sampler substantially improves mixing over the single-move alternative by avoiding the label-switching problem through appropriate identifiability constraints [46].

Block 4: Generator Matrix Q

Given the sampled regime sequence $Z = \{Z_t\}$, the off-diagonal elements of the generator Q have conjugate Gamma posteriors, obtained by summing the observed transition counts and sojourn times in each regime [31].

Block 5: Diffusion Parameters Θ_{diff}

The diffusion parameters $\mu(s)$, $\kappa(s)$, $\theta(s)$, $\sigma_{-}V(s)$, $\rho(s)$ within each regime are updated via a Metropolis–Hastings step with a random-walk proposal. The acceptance probability is computed using the Euler-discretized complete-data likelihood, which Eraker et al. [10] showed introduces negligible discretization bias at daily observation frequency. A blocking strategy groups parameters with high posterior correlation to improve mixing.

Block 6: Jump Parameters Θ_{jump}

The co-jump parameters $(\lambda(s), \mu_{-}J(s), \sigma_{-}J(s), \rho_{-}J(s), \eta_{-}V(s))$ for each regime are drawn using conjugate updates where available (Gamma for λ , $\eta_{-}V$; Normal for $\mu_{-}J$, $\rho_{-}J$) and a Metropolis–Hastings step for $\sigma_{-}J(s)$. The availability of the augmented jump sizes makes these updates substantially more efficient than marginal likelihood methods.

4.4 Convergence Diagnostics

Convergence is assessed using the Gelman–Rubin potential scale reduction factor (PSRF) across four independent chains initialized from dispersed starting values. We declare convergence when the PSRF falls below 1.05 for all scalar parameters and latent state summaries. Trace plots, autocorrelation functions, and effective sample size (ESS) calculations are reported for all parameters. In our simulation experiments, convergence is typically achieved within 5,000 burn-in iterations; for the empirical application, we use 50,000 iterations with the first 20,000 discarded as burn-in.

4.5 Model Comparison via Bayes Factors

Model selection is performed using the log marginal likelihood estimated by the harmonic mean estimator of Newton and Raftery [36] and, for robustness, the bridge sampling estimator of Meng and Wong [37]. The Bayes factor $BF_{\{12\}} = p(Y \mid M_{-1}) / p(Y \mid M_{-2})$ comparing models M_{-1} and M_{-2} is interpreted using Kass and Raftery's [38] scale: $BF > 150$ constitutes decisive evidence in favor of M_{-1} . We compare the full RS-SVCJ model against five nested alternatives, including the constant-parameter SVCJ, the RS-Heston, the SVCJ with regime-switching jump intensity only, and the single-regime model with a fixed co-jump correlation.

5. Simulation Study

5.1 Design

We conduct a comprehensive Monte Carlo study to assess the finite-sample properties of the MCMC estimator. Data are simulated from the RS-SVCJ model at daily frequency for sample sizes $T \in \{500, 1000, 2500, 5000\}$, corresponding approximately to 2, 4, 10, and 20 years of trading data. For each sample size, $R = 200$ independent replications are generated. The true parameter values are calibrated to be broadly consistent with the posterior estimates obtained in our empirical application: $\kappa(1) = 4.0$, $\theta(1) = 0.015$, $\sigma_V(1) = 0.25$, $\rho(1) = -0.6$, $\lambda(1) = 0.05$ (tranquil regime) and $\kappa(2) = 2.0$, $\theta(2) = 0.045$, $\sigma_V(2) = 0.40$, $\rho(2) = -0.75$, $\lambda(2) = 0.20$ (crisis regime), with co-jump parameters $\rho_{-J}(1) = -0.3$, $\rho_{-J}(2) = -0.8$. The generator matrix has off-diagonal elements $q_{-}\{12\} = 0.02$, $q_{-}\{21\} = 0.08$, implying expected durations of 50 days in regime 1 and 12.5 days in regime 2.

5.2 Results

Table 2 reports the root mean squared error (RMSE) and bias of the posterior mean estimator across replications. Results confirm that all parameters are well-identified and that the RMSE decreases approximately at the expected $T^{-1/2}$ rate. The bias is negligible for all parameters at $T \geq 1000$. The co-jump regression coefficient $\rho_{-J}(s)$ is the most challenging parameter to estimate, with relative RMSE of approximately 18% at $T = 500$ declining to 7% at $T = 5000$. The regime generator elements $q_{-}\{12\}$ and $q_{-}\{21\}$ are estimated with high precision even at short sample sizes, confirming that the FFBS step efficiently extracts information about regime durations from the observed return data.

Parameter	T = 500	T = 1,000	T = 2,500	T = 5,000
$\kappa(1)$	0.312	0.218	0.134	0.092
$\theta(1)$	0.0021	0.0015	0.0009	0.0006
$\sigma_V(1)$	0.041	0.028	0.017	0.011
$\lambda(1)$	0.012	0.008	0.005	0.003

$\rho_{J(1)}$	0.058	0.039	0.024	0.016
$\rho_{J(2)}$	0.143	0.097	0.061	0.041
$\lambda(2)$	0.038	0.026	0.016	0.011
$q_{\{12\}}$	0.006	0.004	0.002	0.002
$q_{\{21\}}$	0.018	0.013	0.008	0.005

Table 2. Root mean squared error (RMSE) of the posterior mean estimator across $R = 200$ replications, for selected parameters.

Regime identification accuracy, measured by the proportion of correctly classified days (in terms of the modal smoothed regime probability), rises from 84% at $T = 500$ to 97% at $T = 5000$. The PSRF falls below 1.05 for 97% of all parameter–replication combinations at $T = 1000$, rising to 100% at $T = 5000$. Effective sample sizes for the most auto correlated parameters ($\kappa(s)$, $\theta(s)$) are above 500 in all cases with 30,000 MCMC iterations post burn-in, confirming adequate mixing of the Gibbs sampler.

5.3 Sensitivity to Prior Specification

We examine robustness to prior specification by repeating the simulation study under three alternative prior configurations: (i) diffuse priors with substantially larger prior variances; (ii) tight priors centered on incorrect values; (iii) a three-regime model estimated with a two-regime specification (model misspecification). Under diffuse priors, RMSE increases by at most 15% relative to the baseline, and bias remains negligible, confirming that the likelihood dominates the prior at realistic sample sizes. Under tight misspecified priors, Bayesian shrinkage towards the incorrect prior values generates visible bias at $T = 500$ that dissipates by $T = 2500$. Under the misspecified two-regime model, Bayes factors strongly favour the correctly-specified three-regime alternative when the true model has three regimes, validating the model selection procedure.

6. Empirical Application

6.1 Data

We apply the RS-SVCJ model to daily log-returns on the S&P 500 index over the period January 3, 2000 to December 31, 2023, comprising $T = 6,047$ trading days.

The sample intentionally spans multiple episodes of market stress the dot-com crash (2000–2002), the global financial crisis (2007–2009), the COVID-19 shock (2020), and the 2022 inflation-driven bear market as well as extended periods of low volatility and strong equity returns. Returns are computed as $y_t = \log(S_t / S_{t-1})$ and interest rates are proxied by the 3-month Treasury bill rate. Descriptive statistics confirm the usual stylized facts: excess kurtosis of 12.4, negative skewness of -0.72 , and strong temporal clustering of large returns. Standard Ljung–Box tests reject the null of no autocorrelation in squared returns at all conventional significance levels, confirming the presence of volatility clustering.

6.2 Parameter Estimates

Table 3 reports the posterior means and 95% credible intervals for the RS-SVCJ model under the two-regime specification. All parameters are identified with high precision. The estimates reveal striking regime-dependence in all components of the model. In the tranquil regime, the mean-reversion speed $\kappa(1) = 3.91$ (95% CI: $[3.12, 4.87]$) is almost twice that in the crisis regime $\kappa(2) = 2.15$ ($[1.60, 2.80]$), indicating that volatility shocks die out more slowly during crises a finding consistent with the empirical literature on volatility regime dependence [21, 26].

The long-run variance targets $\theta(1) = 0.013$ and $\theta(2) = 0.047$ correspond to annualized volatility levels of approximately 11.5% and 21.8% respectively, broadly consistent with the empirical distinction between bull and bear equity market environments. The vol-of-vol $\sigma_V(2) = 0.43$ is substantially higher than $\sigma_V(1) = 0.24$, reflecting greater uncertainty about the volatility path during crises. The leverage correlation is $\rho(1) = -0.57$ and $\rho(2) = -0.72$, confirming the well-known asymmetric volatility effect and its amplification during downturns.

Parameter	Regime 1 (Tranquil)	Regime 2 (Crisis)	Difference (95% CI)
$\kappa(s)$	3.91 $[3.12, 4.87]$	2.15 $[1.60, 2.80]$	1.76* $[0.98, 2.55]$
$\theta(s)$	0.013 $[0.011, 0.016]$	0.047 $[0.038, 0.057]$	-0.034* $[-0.046, 0.018]$

					-0.022]
$\sigma_V(s)$	0.241	[0.205,	0.429	[0.376,	-0.188* [-0.251,
	0.282]		0.490]		-0.130]
$\rho(s)$	-0.573	[-0.618,	-0.718	[-0.761,	-0.145* [-0.204,
	-0.524]		-0.673]		-0.085]
$\lambda(s)$	0.047	[0.032,	0.211	[0.164,	-0.164* [-0.219,
	0.064]		0.264]		-0.107]
$\mu_J(s)$	-0.018	[-0.029,	-0.041	[-0.058,	-0.023* [-0.040,
	-0.007]		-0.025]		-0.005]
$\sigma_J(s)$	0.031	[0.021,	0.058	[0.044,	-0.027* [-0.043,
	0.043]		0.075]		-0.010]
$\rho_J(s)$ [KEY]	-0.29	[-0.41,	-0.84	[-0.94,	-0.55* [-0.69,
	-0.18]		-0.73]		-0.41]
$\eta_V(s)$	28.4	[21.6, 36.8]	11.7	[8.8, 15.4]	16.7* [9.3, 24.8]

Table 3. Posterior means with 95% credible intervals for the RS-SVCJ model. The asterisk (*) denotes that the credible interval for the regime difference excludes zero, indicating statistically significant regime-dependence.

The most striking finding concerns the co-jump regression coefficient: $\rho_{J(1)} = -0.29$ versus $\rho_{J(2)} = -0.84$, a difference whose 95% credible interval $[-0.69, -0.41]$ excludes zero decisively. This confirms our central hypothesis that the correlation between return jumps and volatility jumps is substantially more negative during crisis episodes large downward return jumps are accompanied by proportionally larger volatility jumps in bear markets than in bull markets. This finding has important implications: single-regime models that average over these two states will systematically misprice deep out-of-the-money put options, which are most sensitive to the joint tail of the return–volatility distribution.

6.3 Regime Identification

The smoothed regime probabilities $P(Z_t = 2 \mid Y_{1:T})$ exhibit a clear pattern over the sample period. Crisis regime probabilities rise sharply above 0.9 during four well-defined episodes: the 2001–2002 post-dot-com retrenchment, the September–November 2008 Lehman Brothers crisis period, the February–March 2020 COVID-19 shock, and the September–October 2022 rapid-tightening episode. The average duration of crisis spells, estimated at $1/q_{21} \approx 18$ trading days (about 3.5 weeks), is consistent with the episodic nature of financial crises and the rapid recovery from the 2020 COVID shock. Regime 2 accounts for approximately 12% of all trading days in the sample a fraction consistent with previous estimates of the proportion of days characterized by anomalous return behavior in the literature [10, 25].

6.4 Option Pricing Performance

We evaluate the option pricing performance of the RS-SVCJ model against four benchmarks the SVCJ model (constant parameters), the RS-Heston model (no jumps, regime switching), the Heston model, and the Black–Scholes model using a cross-sectional dataset of S&P 500 European options for the period January 2010 to December 2023. Following standard practice, we calibrate each model daily by minimizing the sum of squared implied volatility errors across available strike-maturity pairs, using the Bayesian posterior distribution for the current volatility and regime probabilities as initial conditions for the optimization.

Table 4 reports the in-sample and out-of-sample root mean squared implied volatility error (RMIV) across moneyness and maturity buckets. The RS-SVCJ model achieves an overall in-sample RMIV of 1.21 volatility points, compared to 1.89 for the SVCJ (a 36% improvement), 2.47 for RS-Heston, 3.14 for Heston, and 5.83 for Black–Scholes. The largest improvements are concentrated in the deep out-of-the-money put options (moneyness $K/S \leq 0.85$) and in long-dated options (maturity above 12 months), which are most sensitive to the crisis-state co-jump structure. Out-of-sample performance mirrors the in-sample results, confirming that the RS-SVCJ improvements reflect genuine model adequacy rather than in-sample overfitting.

Model	OTM Put	ATM	OTM Call	Long-dated	Overall
Black–Scholes	9.14	3.41	4.87	6.52	5.83
Heston	5.32	1.98	3.11	3.87	3.14
RS-Heston	4.01	1.72	2.63	2.94	2.47
SVCJ	2.87	1.31	1.94	2.11	1.89
RS-SVCJ (this paper)	1.43*	1.08*	1.19*	1.31*	1.21*

Table 4. *Root mean squared implied volatility error (volatility points) by moneyness/maturity category. Asterisk (*) denotes statistically significant improvement over SVCJ at the 1% level (Diebold–Mariano test).*

6.5 Model Comparison via Bayes Factors

Table 5 reports log Bayes factors relative to the RS-SVCJ model. The full model is decisively preferred over all nested alternatives, with log Bayes factors ranging from 18.4 (versus SVCJ) to 84.7 (versus Black–Scholes), all exceeding the threshold of 5.0 for decisive evidence on Kass and Raftery's [38] scale. The comparison between RS-SVCJ and SVCJ (log BF = 18.4) provides the strongest direct evidence for the empirical relevance of regime-switching. The comparison between RS-SVCJ and a constrained version in which $\rho_{-J(1)} = \rho_{-J(2)}$ (log BF = 11.2) confirms that regime-dependent co-jump correlation is the primary driver of the model's superior fit.

7. Conclusion

This paper has introduced and estimated the regime-switching stochastic volatility co-jump (RS-SVCJ) model, a flexible framework for joint modelling of asset returns and volatility that allows all distributional features of co-jumps to vary across macroeconomic regimes governed by a hidden Markov chain. The model nests the leading single-regime models in the literature including the SVCJ model of Eraker,

Johannes, and Polson [10] as special cases, and adds a single economically interpretable structural element: the regime-dependence of the co-jump regression coefficient, which captures how the return–volatility jump correlation varies between tranquil and crisis market environments. Our theoretical contribution is the derivation of the conditional characteristic function of the log-asset price under the full RS-SVCJ model, which satisfies a system of coupled Riccati-type ODEs admitting an efficient numerical solution via standard Runge–Kutta methods. The characteristic function enables fast and accurate option pricing via the Carr–Madan FFT or Fang–Oosterlee COS methods, producing a richer implied volatility surface that better captures the empirically observed patterns across strikes and maturities. Our methodological contribution is a data-augmentation Bayesian MCMC estimator that jointly samples all parameters and latent states including the latent regime sequence via FFBS from the exact posterior distribution. The estimator exhibits excellent finite-sample properties in simulation experiments and is applicable to any sample size available in practice.

The empirical application to 24 years of S&P 500 returns yields three principal findings. First, the co-jump regression coefficient is strongly regime-dependent: $\rho_{J(1)} = -0.29$ in the tranquil regime versus $\rho_{J(2)} = -0.84$ in the crisis regime, confirming that return and volatility jumps are far more tightly coupled during periods of market stress. Second, the identified regime sequence aligns naturally with well-known episodes of market dysfunction the 2001–2002 dot-com crash, the 2008–2009 global financial crisis, the 2020 COVID shock, and the 2022 tightening cycle lending economic credibility to the model. Third, the RS-SVCJ model generates substantially smaller implied volatility pricing errors than all nested benchmarks, with the largest improvements for deep out-of-the-money puts and long-dated options precisely the instruments most sensitive to tail risk.

Several extensions suggest themselves for future research. First, the framework can be extended to a multivariate setting, where regime-dependent co-jumps in multiple asset prices generate regime-varying cross-asset dependence structures relevant for portfolio risk management and multi-asset derivative pricing. Second, the regime-switching structure can be combined with self-exciting (Hawkes)

jump intensity processes to capture the clustering of jump arrivals within each regime a feature suggested by recent high-frequency evidence [22, 23]. Third, the model can be enriched with stochastic jump intensity within each regime, following the recent work on bivariate regime-switching models with Poisson co-jump intensity [41]. Fourth, the incorporation of option price data directly into the estimation (joint physical and risk-neutral estimation) would sharpen identification of the risk premia embedded in the model, following the approach suggested by Broadie, Chernov, and Johannes [39]. We leave these extensions to future research.

Appendix A: Proof of Proposition 1 (Conditional Characteristic Function)

We provide a brief sketch of the derivation of the Riccati ODE system in Section 3.2. Define the vector-valued function $\Psi(u, V, s, \tau) = E^{\mathbb{Q}} [e^{u X(T)} \mid X(t) = x, V(t) = V, Z(t) = s]$. By the tower property and the Markov property of (X, V, Z) , Ψ satisfies the following system of regime-coupled PIDEs:

$$\partial \Psi_s / \partial \tau = \mathcal{L}_s \Psi_s + \sum_{j \neq s} q_{sj} (\Psi_j - \Psi_s) + \lambda(s) \int (\Psi_s(V + j_V, x + j_S) - \Psi_s) dF_s(j_S, j_V),$$

where $\mathcal{L}_s$ is the infinitesimal generator of the diffusion component in regime s . Substituting the affine ansatz $\Psi_s = \exp\{A_s + B_s V + u x\}$ and collecting terms linear in V and those independent of V yields the ODE system for (A_s, B_s) stated in Proposition 1. The jump integral reduces to $\Phi_J(u, B_s, s) - 1$ by direct computation under the Normal-Exponential distribution for $(J_S, J_V \mid Z = s)$.

Appendix B: Full Conditional Distributions

For completeness, we list the full conditional distributions for all blocks of the Gibbs sampler. Let Θ_{-k} denote all parameters except those in block k .

B.1 Generator elements. Conditional on the regime sequence Z , the off-diagonal generator elements follow conjugate Gamma posteriors: $q_{ij} \mid Z \sim \text{Gamma}(a_{ij} + n_{ij}, b_{ij} + T_i)$, where n_{ij} is the total number of transitions from regime i to j and T_i is the total time spent in regime i .

B.2 Jump intensity. Conditional on the jump indicators ξ and regime sequence Z , the jump intensity in regime s follows a conjugate Gamma posterior: $\lambda(s) \mid \xi, Z \sim \text{Gamma}(a_\lambda + \sum_t 1_{\{Z_t=s\}} \xi_t, b_\lambda + T_s)$, where $T_s = \sum_t 1_{\{Z_t=s\}}$ is the

number of observations in regime s .

B.3 Mean jump size. Conditional on the jump sizes J , the indicator ξ , the volatility jump sizes $J \setminus V$, and regime Z , the mean log-jump size in regime s follows a Normal posterior: $\mu_{J(s)} \mid J, \xi, Z \sim \text{Normal}(m, \tilde{v})$, with posterior mean and variance computed in closed form from the sufficient statistics of the observed jump residuals.

Conflict of interest

The authors declared no conflict of interest.

Author Contribution

All authors reviewed the results and approved the final version of the manuscript. They are also accountable for the study's integrity.

References

1. Black F, Scholes M. The pricing of options and corporate liabilities. *J Polit Econ*. 1973;81(3):637–654.
2. Merton RC. Theory of rational option pricing. *Bell J Econ Manag Sci*. 1973;4(1):141–183.
3. Merton RC. Option pricing when underlying stock returns are discontinuous. *J Financ Econ*. 1976;3(1–2):125–144.
4. Kou SG. A jump-diffusion model for option pricing. *Manage Sci*. 2002;48(8):1086–1101.
5. Duffie D, Pan J, Singleton K. Transform analysis and asset pricing for affine jump-diffusions. *Econometrica*. 2000;68(6):1343–1376.
6. Andersen TG, Benzoni L, Lund J. An empirical investigation of continuous-time equity return models. *J Finance*. 2002;57(3):1239–1284.
7. Barndorff-Nielsen OE, Shephard N. Econometrics of testing for jumps in financial economics using bipower variation. *J Financ Econom*. 2006;4(1):1–30.
8. Heston SL. A closed-form solution for options with stochastic volatility with applications to bond and currency options. *Rev Financ Stud*. 1993;6(2):327–343.
9. Bates DS. Jumps and stochastic volatility: exchange rate processes implicit in Deutsche mark options. *Rev Financ Stud*. 1996;9(1):69–107.

10. Eraker B, Johannes M, Polson N. The impact of jumps in volatility and returns. *J Finance*. 2003;58(3):1269–1300.
11. Eraker B. Do stock prices and volatility jump? Reconciling evidence from spot and option prices. *J Finance*. 2004;59(3):1367–1403.
12. Hamilton JD. A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica*. 1989;57(2):357–384.
13. Benbachir S, El Alami A, Lakhnati G. Markov-switching threshold stochastic volatility models with regime changes. *AIMS Math*. 2024;9(2):3119–3154.
14. Fu MC, Li B, Wu R, Zhang T. Option pricing under a discrete-time Markov switching stochastic volatility with co-jump model. *arXiv:2006.15054*. 2020.
15. Shirzadi M, Rostami M, Dehghan M, Li X. American options pricing under regime-switching jump-diffusion models with meshfree finite point method. *Chaos Solitons Fractals*. 2023;166:C.
16. Chen Y. Fast and accurate computation of the regime-switching jump-diffusion option prices using Laplace transform and compact difference with convergence guarantee. *Comput Econ*. 2024;64(1):57–80.
17. Jacod J, Todorov V. Testing for common arrivals of jumps for discretely observed multidimensional processes. *Ann Stat*. 2009;37(4):1792–1838.
18. Bibinger M, Winkelmann L. Econometrics of co-jumps in high-frequency data with noise. *J Econom*. 2015;184(2):361–378.
19. Caporin M, Kolokolov A, Renò R. Systemic co-jumps. *J Financ Econ*. 2017;126(3):563–591.
20. Andersen TG, Bollerslev T, Diebold FX. Roughing it up: including jump components in the measurement, modeling, and forecasting of return volatility. *Rev Econ Stat*. 2007;89(4):701–720.
21. Bollerslev T, Todorov V. Tails, fears and risk premia. *J Finance*. 2011;66(6):2165–2211.
22. Dai Z, Chang X. Co-jumps in the Chinese stock market before, during and after COVID-19. *Finance Res Lett*. 2024; doi:10.1016/j.frl.2024.105231.
23. Liao Y, Anderson HM. Co-jumps, co-jump tests, and volatility forecasting: Monte Carlo and empirical evidence. *J Risk Financ Manag*. 2022;15(8):334.

24. Caldana R, Fusai G, Göncü A, Peressini A. Spread option pricing in regime-switching jump diffusion models. *Mathematics*. 2022;10(9):1574.
25. Carverhill A, Luo D. A Bayesian analysis of time-varying jump risk in S&P 500 returns and options. *J Financ Mark*. 2023;64:C.
26. Goswami A, Mukherjee KN, Patalwala IH, Sanjay NS. Regime recovery using implied volatility in Markov modulated market model. *arXiv:2201.10304*. 2022.
27. Ma C, Xu W, Dong Y. Forward starting options pricing under a regime-switching jump-diffusion model with Wishart stochastic volatility and stochastic interest rate. *Int J Comput Math*. 2024;101(3):331–356.
28. Carr P, Madan D. Option valuation using the fast Fourier transform. *J Comput Finance*. 1999;2(4):61–73.
29. Fang F, Oosterlee CW. A novel pricing method for European options based on Fourier-cosine series expansions. *SIAM J Sci Comput*. 2008;31(2):826–848.
30. Chen Y, Li L. An efficient IMEX compact scheme for the coupled time fractional integro-differential equations arising from option pricing with jumps. *Comput Econ*. 2025;65(4):2397–2422.
31. Kim CJ, Nelson CR. *State-Space Models with Regime Switching: Classical and Gibbs-Sampling Approaches with Applications*. Cambridge (MA): MIT Press; 1999.
32. He XJ, Lin S. Analytical formulae for variance and volatility swaps with stochastic volatility, stochastic equilibrium level and regime switching. *AIMS Math*. 2024;9(8):22225–22238.
33. Stumpf-Fétizon T, Łatuszyński K, Palczewski J, Roberts G. Exact Bayesian inference for Markov switching diffusions. *arXiv:2502.09126*. 2025.
34. Gonçalves S, Guidolin M. Predictable dynamics in the S&P 500 index options implied volatility surface. *J Bus*. 2006;79(3):1591–1635.
35. Lindsten F, Jordan MI, Schön TB. Particle Gibbs with ancestor sampling. *J Mach Learn Res*. 2014;15(1):2145–2184.
36. Newton MA, Raftery AE. Approximate Bayesian inference with the weighted

- likelihood bootstrap. *J R Stat Soc Ser B*. 1994;56(1):3–48.
37. Meng XL, Wong WH. Simulating ratios of normalizing constants via a simple identity: a theoretical exploration. *Stat Sin*. 1996;6(4):831–860.
 38. Kass RE, Raftery AE. Bayes factors. *J Am Stat Assoc*. 1995;90(430):773–795.
 39. Broadie M, Chernov M, Johannes M. Model specification and risk premia: evidence from futures options. *J Finance*. 2007;62(3):1453–1490.
 40. Johannes M, Polson N. MCMC methods for continuous-time financial econometrics. In: Aït-Sahalia Y, Hansen L, editors. *Handbook of Financial Econometrics*. Amsterdam: North-Holland; 2009. p. 1–72.
 41. Ma C, Xu W. Some bivariate options pricing in a regime-switching stochastic volatility jump-diffusion model with stochastic intensity, stochastic interest and dependent jump. *Math Comput Simul*. 2024; doi:10.1016/j.matcom.2024.10.023.
 42. Golomoziy V, Kladivko K, Mishura Y. Discrete-time weak approximation of a Black-Scholes model with drift and volatility Markov switching. *arXiv:2501.06895*. 2025.
 43. De Vries T. Beyond Carr–Madan: a projection approach to risk-neutral moment estimation. *arXiv:2601.14852*. 2026.
 44. Bauwens L, Dufays A, Rombouts JVK. Marginal likelihood for Markov-switching and change-point GARCH models. *J Econom*. 2014;178(3):508–522.
 45. Carter CK, Kohn R. On Gibbs sampling for state space models. *Biometrika*. 1994;81(3):541–553.
 46. Frühwirth-Schnatter S. *Finite Mixture and Markov Switching Models*. New York: Springer; 2006.