Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about

Volume 4, Issue 5 (2026)

Online ISSN 3007-3197 | Print ISSN 3007-3189

http://amresearchreview.com/index.php/Journal/about

A Note On Double-Null Form For Space-Times Metrices Having (2 + 1) And (3 + 1) Dimensions

S. M. Jawwad Riaz
[1]Department of Mathematics, COMSATS University Islamabad, Wah Campus, G.T. Road, Wah Cantt. 47040, Pakistan. jawwadriaz@yahoo.com

A. A. Siddiqui
[2]School of Natural Sciences, National University of Sciences and Technology,H-12 Islamabad, 46000, Pakistan. azad@sns.nust.edu.pk

ABSTRACT

In this research article, (2 + 1) dimension space-times are categorized, depending on the probability of transforming into a double-null form. Results orig-inate that those classes of (2 + 1)−dimensional space-times that have non-zero co-efficients, g_{02} or g_{12}, are unable to transform to a desirable form. Further, it is also confirmed that a class of (3 + 1)−dimensional space-times possessing non-zero coefficients, g_{02} or g_{12}, cannot be transformed to a double-null form.

Keywords— Double-Null Form, Coordinate Transformation, Space-Time Metrices.

1 INTRODUCTION

A variety of different forms of space-time metrices are castoff in, solar-terrestrial relationships, exploration of black hole (BH) space-times, Numerical Relativity and Newman-Penrose formal-ism etc. The necessity for other forms stand up from the datum that often numerous physical procedures are more implicit, data under investigation is well-organized or calculations with more ease, in a particular form then in another one. In literature of relativity, a double-null (DN) form in (2 + 1) dimensional space-times has a form:

$$ds^2 = 2\tilde{g}_{01}\, d\hat{u}\, d\hat{v} + \tilde{g}_{22}\, d\phi^2, \qquad (1)$$

where $\tilde{g}_{01}$ and $\tilde{g}_{22}$ are functions of $\hat{u}$, $\hat{v}$ and ϕ.

It is extensively utilized by Howard in exploring BH space-times [1] and by Roman in their prior work [2]. It is also supportive in Newman-Penrose formalism [3]−[6]. In the study of vac-uum space-times, the Newman-Penrose formalism make things easier in DN form, that permits

Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about

Volume 4, Issue 5 (2026)

Online ISSN 3007-3197 Print ISSN 3007-3189

http://amresearchreview.com/index.php/Journal/about

straightforward proof of numerous theorems.
In Numerical Relativity, DN form play a major role that applied to various zones, for instance in perturbed BH's, cosmological models, and neutron stars [7]–[9]. In numerical analysis, vigilant courtesy is considered for the stability and convergence of numerical solutions and is possible only with DN form. Apart from Numerical Relativity and Newman-Penrose formalism, it has been useful in formulation of light-cone gauge of string theory, that is a remarkable effort to unite quantum physics with relativity [10]–[14].
The plan for the research paper is, in sections two and three, it is focused on the existence of DN form for different classes of (2 + 1) and (3 + 1) dimensional space-times. Where as Section 4, is reserved for conclusion and discussion.

2 EXISTENCE OF DOUBLE-NULL FORM FOR (2+1) DI-MENSIONAL SPACE-TIMES

The (2 + 1) dimensional space-time is expressed as metric as

$$ds^2 = g_{00}dt^2 + g_{11}dr^2 + g_{22}d\phi^2 + 2g_{01}dtdr + 2g_{02}dtd\phi + 2g_{12}drd\phi\,, \qquad (2)$$

here g_{00}, g_{11}, g_{22}, g_{01}, g_{02} and g_{12} are function of t, r and ϕ.
For transformation of a metric (2) to a desired form (1), let t and r be a function of $\hat{u}$, $\hat{v}$ and utilizing transformation of coordinates:

$$g_{\tilde a\tilde b} = \frac{\partial x^p}{\partial x^{\tilde a}}\cdot\frac{\partial x^q}{\partial x^{\tilde b}}\cdot g_{lm}\,, \qquad (3)$$

where x^p, x^q and $x^{\tilde a}$, $x^{\tilde b}$ corresponds to the (t, r, ϕ) and $(\hat{u}, \hat{v}, \phi)$ coordinates respectively, with $p, q, \tilde a, \tilde b = 0, 1, 2$, to get the partial differential equations (PDE) system as

$$g_{\tilde 0\tilde 0} = \left(\frac{\partial t}{\partial \hat u}\right)^2 g_{00} + \left(\frac{\partial r}{\partial \hat u}\right)^2 g_{11} + 2\left(\frac{\partial t}{\partial \hat u}\right)\left(\frac{\partial r}{\partial \hat u}\right) g_{01}, \qquad (4)$$

$$g_{\tilde 1\tilde 1} = \left(\frac{\partial t}{\partial \hat v}\right)^2 g_{00} + \left(\frac{\partial r}{\partial \hat v}\right)^2 g_{11} + 2\left(\frac{\partial t}{\partial \hat v}\right)\left(\frac{\partial r}{\partial \hat v}\right) g_{01}, \qquad (5)$$

Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about
Volume 4, Issue 5 (2026)
Online ISSN 3007-3197
Print ISSN 3007-3189
http://amresearchreview.com/index.php/Journal/about

$$g2\tilde{}2 = g_{22}, \qquad (6)$$

Online ISSN 3007-3197 | Print ISSN 3007-3189
http://amresearchreview.com/index.php/Journal/about

$$g_{\tilde 0 \tilde 1} = \left(\frac{\partial t}{\partial \hat u}\right)\left(\frac{\partial t}{\partial \hat v}\right) g_{00} + \left(\frac{\partial r}{\partial \hat u}\right)\left(\frac{\partial r}{\partial \hat v}\right) g_{11} + \left(\frac{\partial t}{\partial \hat u}\frac{\partial r}{\partial \hat v} + \frac{\partial r}{\partial \hat u}\frac{\partial t}{\partial \hat v}\right) g_{01}, \qquad (7)$$

$$g_{\tilde 0 \tilde 2} = \left(\frac{\partial t}{\partial \hat u}\right) g_{02} + \left(\frac{\partial r}{\partial \hat u}\right) g_{12}, \qquad (8)$$

$$g_{\tilde 1 \tilde 2} = \left(\frac{\partial t}{\partial \hat v}\right) g_{02} + \left(\frac{\partial r}{\partial \hat v}\right) g_{12}. \qquad (9)$$

With the required condition: $g_{\tilde 0 \tilde 0} = g_{\tilde 1 \tilde 1} = g_{\tilde 0 \tilde 2} = g_{\tilde 1 \tilde 2} = 0$, the above set of PDE's from (4) to (9) condense as

$$\left(\frac{\partial t}{\partial \hat u}\right)^2 g_{00} + \left(\frac{\partial r}{\partial \hat u}\right)^2 g_{11} + 2\left(\frac{\partial t}{\partial \hat u}\right)\left(\frac{\partial r}{\partial \hat u}\right) g_{01} = 0, \qquad (10)$$

$$\left(\frac{\partial t}{\partial \hat v}\right)^2 g_{00} + \left(\frac{\partial r}{\partial \hat v}\right)^2 g_{11} + 2\left(\frac{\partial t}{\partial \hat v}\right)\left(\frac{\partial r}{\partial \hat v}\right) g_{01} = 0, \qquad (11)$$

$$g_{22} = g_{\tilde 2 \tilde 2}, \qquad (12)$$

$$\left(\frac{\partial t}{\partial \hat u}\right)\left(\frac{\partial t}{\partial \hat v}\right) g_{00} + \left(\frac{\partial r}{\partial \hat u}\right)\left(\frac{\partial r}{\partial \hat v}\right) g_{11} + \left(\frac{\partial t}{\partial \hat u}\frac{\partial r}{\partial \hat v} + \frac{\partial r}{\partial \hat u}\frac{\partial t}{\partial \hat v}\right) g_{01} = g_{\tilde 0 \tilde 1}, \qquad (13)$$

$$\frac{\partial t}{\partial \hat u} g_{02} \qquad \left(\frac{\partial t}{}\right) g + \frac{\partial r}{\partial \hat u} g_{12}$$

Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about

Volume 4, Issue 5 (2026)

Online ISSN 3007-3197
Print ISSN 3007-3189

http://amresearchreview.com/index.php/Journal/about

= 0,

$+ \left(\frac{\partial r}{\partial \hat{v}} \right) g_{02}$ (14)

= 0. (15)

$\partial \hat{v}$ 02 $\partial \hat{v}$ 12

The desired Jacobian is,

$$J = \left(\frac{\partial t}{\partial \hat{u}} \right) \left(\frac{\partial r}{\partial \hat{v}} \right) - \left(\frac{\partial t}{\partial \hat{v}} \right) \left(\frac{\partial r}{\partial \hat{u}} \right), \quad (16)$$

for a non-zero transformation (3), Eq.(14) and (15) possess only trivial solution $g_{02} = g_{12} = 0$

Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about

Volume 4, Issue 5 (2026)

Online ISSN 3007-3197
Print ISSN 3007-3189
http://amresearchreview.com/index.php/Journal/about

. It reflects that if a metric (2) comprises g_{02} or g_{12} then it could not be transformed to the desired DN form (1). Thus, the utmost general form for the (2 + 1) dimensional space-time metric that can be transformed to a DN form is

$$ds^2 = g_{00}dt^2 + g_{11}dr^2 + g_{22}d\phi^2 + 2g_{01}dtdr. \qquad (17)$$

In order to obtain specific transformations to transform metric (17) in double null form, we consider the following cases:
(a) $g_{00}, g_{11}, g_{22} \neq 0$ and $g_{01} = 0$
(d) $g_{00}, g_{22}, g_{01} \neq 0$ and $g_{11} = 0$
(c) $g_{11}, g_{22}, g_{01} \neq 0$ and $g_{00} = 0$
(d) $g_{00}, g_{11}, g_{22}, g_{01} \neq 0$.

2.1 Case a: $g_{00}, g_{11}, g_{22} \neq 0$ and $g_{01} = 0$

The above set of Eqs.(10) to (13) becomes

$$g_{00}\left(\frac{\partial t}{\partial \hat u}\right)^2 + g_{11}\left(\frac{\partial r}{\partial \hat u}\right)^2 = 0, \quad (18)$$

$$g_{00}\left(\frac{\partial t}{\partial \hat v}\right)^2 + g_{11}\left(\frac{\partial r}{\partial \hat v}\right)^2 = 0, \quad (19)$$

$$g_{22} = \tilde g_{22}, \qquad (20)$$

$$g_{00}\left(\frac{\partial t}{\partial \hat u}\right)\left(\frac{\partial t}{\partial \hat v}\right) + g_{11}\left(\frac{\partial r}{\partial \hat u}\right)\left(\frac{\partial r}{\partial \hat v}\right) = \tilde g_{01}. \qquad (21)$$

With the requirement of non-zero Jacobian (16), Eqs.(18) and (19) implies

$$\left(\frac{\partial t}{\partial \hat u}\right)\left(\frac{\partial r}{\partial \hat v}\right) + \left(\frac{\partial t}{\partial \hat v}\right)\left(\frac{\partial r}{\partial \hat u}\right) = 0. \quad (22)$$

Which results in the transformations of

Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about

Volume 4, Issue 5 (2026)

Online ISSN 3007-3197 Print ISSN 3007-3189
http://amresearchreview.com/index.php/Journal/about

$$t = \Phi(\hat{u} \pm \hat{v}), \; r = \Psi(\hat{u} \mp \hat{v}), \quad (23)$$

with Φ and Ψ as arbitrary functions.

Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about
Volume 4, Issue 5 (2026)

Online ISSN 3007-3197
Print ISSN 3007-3189
http://amresearchreview.com/index.php/Journal/about

2.2 Case b: $g_{00}, g_{22}, g_{01} \neq 0$ and $g_{11} = 0$

While considering the case b, the above system from Eqs.(10) to (13) becomes as

$$\left(\frac{\partial t}{\partial \hat{u}}\right)^2 g_{00} + 2\left(\frac{\partial t}{\partial \hat{u}}\right)\left(\frac{\partial r}{\partial \hat{u}}\right) g_{01} = 0, \quad (24)$$

$$\left(\frac{\partial t}{\partial \hat{v}}\right)^2 g_{00} + 2\left(\frac{\partial t}{\partial \hat{v}}\right)\left(\frac{\partial r}{\partial \hat{v}}\right) g_{01} = 0, \quad (25)$$

$$g_{22} = \tilde{g}_{22}, \quad (26)$$

$$\left(\frac{\partial t}{\partial \hat{u}}\right)\left(\frac{\partial t}{\partial \hat{v}}\right) g_{00} + \left(\frac{\partial t}{\partial \hat{u}}\frac{\partial r}{\partial \hat{v}} + \frac{\partial r}{\partial \hat{u}}\frac{\partial t}{\partial \hat{v}}\right) g_{01} = \tilde{g}_{01}. \quad (27)$$

Eqs.(24) and (25) can be expressed as

$$\frac{\partial t}{\partial \hat{u}}\left(\frac{\partial t}{\partial \hat{u}} g_{00} + 2\frac{\partial r}{\partial \hat{u}} g_{01}\right) = 0 \quad (28)$$

and

$$\frac{\partial t}{\partial \hat{v}}\left(\frac{\partial t}{\partial \hat{v}} g_{00} + 2\frac{\partial r}{\partial \hat{v}} g_{01}\right) = 0. \quad (29)$$

Eqs.(28) and (29) are satisfied by the cases:

$(b1)$ $\frac{\partial t}{\partial \hat{u}} = \frac{\partial t}{\partial \hat{v}} =$

$(b2)$ $\frac{\partial t}{\partial \hat{u}} g_{00} + 2\frac{\partial r}{\partial \hat{u}} g_{01} = \frac{\partial t}{\partial \hat{v}} g_{00} + 2\frac{\partial r}{\partial \hat{v}} g_{01} = 0$

Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about

Volume 4, Issue 5 (2026)

Online ISSN 3007-3197 Print ISSN 3007-3189
http://amresearchreview.com/index.php/Journal/about

$(b3)\ \frac{\partial t}{\partial \hat{u}} = \frac{\partial t}{\partial \hat{v}} g_{00} + 2 \frac{\partial r}{\partial \hat{v}} g_{01} = 0$

$(b4)\ \frac{\partial t}{\partial \hat{v}} = \frac{\partial t}{\partial \hat{u}} g_{00} + 2 \frac{\partial r}{\partial \hat{u}} g_{01} = 0.$

It is observed that in cases $(b1)$ and $(b2)$, the required DN form does not exist as it lead to contradictions $\tilde{g}_{01} = 0$ and $g_{00} = 0$. Anyhow, required DN form exist for the 2 cases, $(b3)$ and $(b4)$.

2.3 Case c: $g_{11}, g_{22}, g_{01} \neq 0$ and $g_{00} = 0$

In case c, the system of Eqs.(10) to (13) compiles to

$$\left(\frac{\partial r}{\partial \hat{u}}\right)^2 g_{11} + 2\left(\frac{\partial t}{\partial \hat{u}}\right)\left(\frac{\partial r}{\partial \hat{u}}\right) g_{01} = 0, \quad (30)$$

$$\left(\frac{\partial r}{\partial \hat{v}}\right)^2 g_{11} + 2\left(\frac{\partial t}{\partial \hat{v}}\right)\left(\frac{\partial r}{\partial \hat{v}}\right) g_{01} = 0, \quad (31)$$

$$g_{22} = \tilde{g}_{22}, \quad (32)$$

$$\left(\frac{\partial r}{\partial \hat{u}}\right)\left(\frac{\partial r}{\partial \hat{v}}\right) g_{11} + \left(\frac{\partial t}{\partial \hat{u}}\frac{\partial r}{\partial \hat{v}} + \frac{\partial r}{\partial \hat{u}}\frac{\partial t}{\partial \hat{v}}\right) g_{01} = \tilde{g}_{01}. \quad (33)$$

Eqs.(29) and (30) can be expressed as

$$\frac{\partial r}{\partial \hat{u}}\left(\frac{\partial r}{\partial \hat{u}} g_{11} + 2\frac{\partial t}{\partial \hat{u}} g_{01}\right) = 0 \quad (34)$$

Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about
Volume 4, Issue 5 (2026)
Online ISSN 3007-3197
Print ISSN 3007-3189
http://amresearchreview.com/index.php/Journal/about

and

$\underline{\partial r}$ ($\underline{\partial r g}$ $\underline{\partial t}$) = 0. (35)
+ 2

g

$\partial \hat{v}$ $\partial \hat{v}$ ^{11} $\partial \hat{v}$ ^{01}

Eqs.(34) and (35), satisfied the following cases: $(c1)$ $\frac{\partial r}{\partial \hat{u}} = \frac{\partial r}{\partial \hat{v}} = 0$

$(c2)$ $\frac{\partial r}{\partial \hat{u}} g11 + 2 \frac{\partial t}{\partial \hat{u}} g01 = \frac{\partial r}{\partial \hat{v}} g11 + 2 \frac{\partial t}{\partial \hat{v}} g01 = 0$

$(c3)$ $\frac{\partial r}{\partial \hat{u}} = \frac{\partial r}{\partial \hat{v}} g11 + 2 \frac{\partial t}{\partial \hat{v}} g01 = 0$

$(c4)$ $\frac{\partial r}{\partial \hat{v}} = \frac{\partial r}{\partial \hat{u}} g11 + 2 \frac{\partial t}{\partial \hat{u}} g01 = 0.$

Cases $(c1)$ and $(c2)$, again lead to contradictions $g_{\tilde{0}1} = 0$ and $g_{11} = 0$ respectively, anyhow, the form exist for the cases $(c3)$ and $(c4)$.

2.4 Case d: $g_{00}, g_{11}, g_{22}, g_{01} \neq 0$

In considering case d, the above system of Eqs.(10) to (13) compiles as

$$\left(\frac{\partial t}{\partial \hat{u}}\right)^2 g_{00} + \left(\frac{\partial r}{\partial \hat{u}}\right)^2 g_{11} + 2 \left(\frac{\partial t}{\partial \hat{u}}\right)\left(\frac{\partial r}{\partial \hat{u}}\right) g_{01} = 0, (36)$$

$$\left(\frac{\partial t}{\partial \hat{v}}\right)^2 g_{00} + \left(\frac{\partial r}{\partial \hat{v}}\right)^2 g_{11} + 2 \left(\frac{\partial t}{\partial \hat{v}}\right)\left(\frac{\partial r}{\partial \hat{v}}\right) g_{01} = 0, (37)$$

$g_{22} = g_{\tilde{2}\tilde{2}}$ $\left(\underline{\partial t}\right)\left(\underline{\partial t}\right) g_{00}$ $+ \left(\underline{\partial r}\right)\left(\underline{\partial r}\right) g$ $+ \left(\underline{\partial t}\,\underline{\partial r} + \underline{\partial r}\,\underline{\partial t}\right) g$
$= g$

Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about
Volume 4, Issue 5 (2026)

Online ISSN 3007-3197 Print ISSN 3007-3189
http://amresearchreview.com/index.php/Journal/about

. (39)

$\partial\hat{u}$ $\partial\hat{v}$ $\partial\hat{u}$ $\partial\hat{v}$ $\partial\hat{u}_{11}\partial\hat{v}\,\partial\hat{u}\,\partial\hat{v}$ $0\tilde{}1_{01}$

Requiring the Jacobian (16) to be non-zero, Eqs.(36) and (37) imply

$$\left(\frac{\partial t}{\partial\hat{u}}\right)\left(\frac{\partial t}{\partial\hat{v}}\right)g_{00} - \left(\frac{\partial r}{\partial\hat{u}}\right)\left(\frac{\partial r}{\partial\hat{v}}\right)g_{11} = 0, \quad (40)$$

$$\left(\frac{\partial t}{\partial\hat{u}}\frac{\partial r}{\partial\hat{v}} + \frac{\partial t}{\partial\hat{v}}\frac{\partial r}{\partial\hat{u}}\right)g_{11} + 2\left(\frac{\partial t}{\partial\hat{u}}\right)\left(\frac{\partial t}{\partial\hat{v}}\right)g_{01} = 0, \quad (41)$$

$$\left(\frac{\partial t}{\partial\hat{u}}\frac{\partial r}{\partial\hat{v}} + \frac{\partial t}{\partial\hat{v}}\frac{\partial r}{\partial\hat{u}}\right)g_{00} + 2\left(\frac{\partial r}{\partial\hat{u}}\right)\left(\frac{\partial r}{\partial\hat{v}}\right)g_{01} = 0. \quad (42)$$

The above system of Eqs.(40) to (42) have non-trivial solutions if

$$\left(\frac{\partial t}{\partial\hat{u}}\frac{\partial r}{\partial\hat{v}} + \frac{\partial t}{\partial\hat{v}}\frac{\partial r}{\partial\hat{u}}\right) = 0, \quad (43)$$

which results in transformations:

$t = \Phi(\hat{u} \pm \hat{v}),\ r = \Psi(\hat{u} \mp \hat{v})$, (44) where Φ and Ψ are arbitrary functions.

3 DOUBLE-NULL FORM FOR (3+1) DIMENSIONAL SPACE-TIMES

Consider the general four dimensional space-time metric:

$$ds^2 = g_{00}dt^2+g_{11}dr^2+g_{22}d\theta^2+g_{33}d\phi^2+2g_{01}dtdr+2g_{02}dtd\theta+2g_{03}dtd\phi+2g_{12}drd\theta+2g_{13}drd\phi+2g_{23}d\theta d\phi, \quad (45)$$

where g_{00}, g_{11}, g_{22}, g_{33}, g_{01}, g_{02}, g_{03}, g_{12}, g_{13} and g_{23} are functions of t, r, θ and ϕ.
For the purpose of transformation of metric (45) to a desired DN form (1), take t and r to be function of $(\hat{u}, \hat{v})$ and utilize again the transformation,

$$g_{\tilde{a}\tilde{b}} = \frac{\partial x^p}{\partial x^{\tilde{a}}}\frac{\partial x^q}{\partial x^{\tilde{b}}}g_{pq}.$$

Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about
Volume 4, Issue 5 (2026)
Online ISSN 3007-3197
Print ISSN 3007-3189
http://amresearchreview.com/index.php/Journal/about

$.g_{lm}$, (46)
where x^p, x^q and x^a, x^b refer to the (t, r, θ, ϕ) and $(\hat{u}, \hat{v}, \theta, \phi)$ coordinates respectively, with p, q, a, b = 0, 1, 2, 3, and also requiring $g00 = g11 = g22 = g02 = g03 = g12 = g13 = g23 = 0$, to get the

Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about
Volume 4, Issue 5 (2026)
Online ISSN 3007-3197
Print ISSN 3007-3189
http://amresearchreview.com/index.php/Journal/about

, (38)

Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about
Volume 4, Issue 5 (2026)

Online ISSN 3007-3197
Print ISSN 3007-3189

http://amresearchreview.com/index.php/Journal/about

system of PDE's as:

$$\left(\frac{\partial t}{\partial \hat{u}}\right)^2 g_{00} + \left(\frac{\partial r}{\partial \hat{u}}\right)^2 g_{11} + 2\left(\frac{\partial t}{\partial \hat{u}}\right)\left(\frac{\partial r}{\partial \hat{u}}\right) g_{01} = 0, \quad (47)$$

$$\left(\frac{\partial t}{\partial \hat{v}}\right)^2 g_{00} + \left(\frac{\partial r}{\partial \hat{v}}\right)^2 g_{11} + 2\left(\frac{\partial t}{\partial \hat{v}}\right)\left(\frac{\partial r}{\partial \hat{v}}\right) g_{01} = 0, \quad (48)$$

$$g_{22} = 0, \quad (49)$$

$$g_{33} = g_{\tilde{3}\tilde{3}}, \quad (50)$$

$$\left(\frac{\partial t}{\partial \hat{u}}\right)\left(\frac{\partial t}{\partial \hat{v}}\right) g_{00} + \left(\frac{\partial r}{\partial \hat{u}}\right)\left(\frac{\partial r}{\partial \hat{v}}\right) g_{11} + \left(\frac{\partial t}{\partial \hat{u}}\frac{\partial r}{\partial \hat{v}} + \frac{\partial r}{\partial \hat{u}}\frac{\partial t}{\partial \hat{v}}\right) g_{01} = g_{\tilde{0}\tilde{1}}, \quad (51)$$

$$\left(\frac{\partial t}{\partial \hat{u}}\right) g_{02} + \left(\frac{\partial r}{\partial \hat{u}}\right) g_{12} = 0, \quad (52)$$

$$\left(\frac{\partial t}{\partial \hat{v}}\right) g_{02} + \left(\frac{\partial r}{\partial \hat{v}}\right) g_{12} = 0. \quad (53)$$

$$\left(\frac{\partial t}{\partial \hat{u}}\right) g_{03} + \left(\frac{\partial r}{\partial \hat{u}}\right) g_{13} = 0, \quad (54)$$

$$\left(\frac{\partial t}{\partial \hat{v}}\right) g_{03} + \left(\frac{\partial r}{\partial \hat{v}}\right) g_{13} = 0. \quad (55)$$

Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about
Volume 4, Issue 5 (2026)

Online ISSN 3007-3197
Print ISSN 3007-3189
http://amresearchreview.com/index.php/Journal/about

$g_{23} = 0,$(56)

Requiring the Jacobian to be:

$$J = \left(\frac{\partial t}{\partial \hat{u}}\right)\left(\frac{\partial r}{\partial \hat{v}}\right) - \left(\frac{\partial t}{\partial \hat{v}}\right)\left(\frac{\partial r}{\partial \hat{u}}\right) \quad (57)$$

of the transformation (46) to be non-zero, Eqs. (52) to (56), have only trivial solution $g_{02} = g_{12} = g_{13} = g_{03} = g_{23} = 0$. It predicts that if the metric (45) have g_{02} or g_{12} then it is not possible to transformed into the desired form (1).

4 CONCLUSION

Utilizing coordinate transformations, DN form for distinct space-times metrices in (2 + 1) and (3+1) dimensions have been formed. For that, it is presented the existence of DN form for differ-ent classes of general space-times metric in (2 + 1) dimensions and is observed that required DN form existence depends only on two coefficients g_{02} and g_{12} of the metric. A class of (2 + 1) di-mensional space-time possessing non-zero coefficients g_{02} or g_{12} , it is not possible to transformed to the desired form. For the remaining classes, further investigation showed that a class in which $g_{00}, g_{11}, g_{22} \neq 0$ and $g_{01} = 0$ can be transformed into the required DN form. However, a class in which $g_{00}, g_{22}, g_{01} \neq 0$ and $g_{11} = 0$, having $\frac{\partial t}{\partial \hat{u}} = \frac{\partial t}{\partial \hat{v}} = 0$ or $\frac{\partial t}{\partial \hat{u}} g_{00} + 2\frac{\partial r}{\partial \hat{u}} g_{01} = \frac{\partial t}{\partial \hat{v}} g_{00} + 2\frac{\partial r}{\partial \hat{v}} g_{01} = 0$ cannot be transformed to DN form. Similar situation arises in a class which has $g_{11}, g_{22}, g_{01} \neq 0$ and $g_{00} = 0$, having $\frac{\partial r}{\partial \hat{u}} = \frac{\partial r}{\partial \hat{v}} = 0$ or $\frac{\partial r}{\partial \hat{u}} g_{11} + 2\frac{\partial t}{\partial \hat{u}} g_{01} = \frac{\partial r}{\partial \hat{v}} g_{11} + 2\frac{\partial t}{\partial \hat{v}} g_{01} = 0$. For a class in which $g_{00}, g_{11}, g_{22}, g_{01} \neq 0$, can be transformed into the DN form.

It is further proved that the existence of DN form depends on two coefficients g_{02} and g_{12}. A class of $(3 + 1)-$dimensional space-times in which coefficients g_{02} or g_{12} are non-zero, cannot be transformed to the DN form. It will be interesting to adopt the procedure used in section 2, to obtain the DN form for different classes of $(3 + 1)-$dimension space-times.

Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about
Volume 4, Issue 5 (2026)

Online ISSN 3007-3197
Print ISSN 3007-3189
http://amresearchreview.com/index.php/Journal/about

References

S. A. Hayward, Phys. Rev. D 49,831 (1994).
T. A. Roman and P. G. Bergmann, Phys. Rev. D 28,1265(1983).
G. Ludwig and S. B. Edgar, Gen. Rel. Grav. 34,807835(20020.
A. Pravdova and V. Pravda, Class. Quantum Grav.25,235008(2008).
A. Papapetrou, Comm. Math. Phys. 34,229(1973).
Y. Sucu and N. Unal, Class. Quantum Grav.6,1443(2004).
R. W. Corkill and J. M. Stewart, Phys.Rev. D 67,373-391(2003).
F. Girotto and A. Saa, Phys. Rev. D70,084014(2004).
V. Husain, Phys. Rev. D52,6860(1995).
Y. Baba, et al., JHEP 12,010 (2009).
I. Nikitin and L. Nikitina ; arXiv:hep-th/0306010v1.
I. N. Nikitin ; arXiv:hep-th/9906003v1.
P. Lee, S. Moriyama, and J. W. Park, Phys. Rev. D 66,085021(2002).
M. Torabian, Phys. Rev. D 76,026006(2007).