

A Complex Neutrosophic Soft Rough Framework for Efficient Multi-Attribute Decision-Making Under Uncertainty

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Abstract. Decision-making plays a vital role across diverse domains including risk assessment, engineering, medical diagnosis, artificial intelligence, and machine learning particularly in environments characterized by incomplete, inconsistent information. Within the health care industry, especially in medical diagnosis, selecting the most suitable alternative based on multiple attributes presents a significant challenge. Conventional decision-making frameworks grounded in classical set theory such as fuzzy sets and soft sets often fall short when addressing highly complex and uncertain scenarios. To overcome these limitations, this study presents an advanced model based on complex neutrosophic soft set with roughness structures. This approach combines the strengths of neutrosophic sets, soft sets, and rough sets within a complex domain thereby improving the handling of uncertainty and enhancing decision-making accuracy. In addition, a novel decision-making algorithm is developed within this framework to streamline the selection process by reducing redundant computations and efficiently identifying optimal alternatives. Comparative analysis with existing models and algorithms highlights the validity and superior performance of the proposed framework.

Keywords: Complex neutrosophic soft rough sets; multi-attribute decision-making; medical diagnosis; uncertainty modeling; rough approximations; soft set theory; decision-making algorithm; comparative analysis.

1. Introduction

Decision making is a fundamental activity in every field of human endeavors across areas such as risk assessment, medical diagnosis, engineering design, financial forecasting, management sciences and industrial planning. In these domains, the quality of outcomes is highly dependent on the accuracy of decision making. Real world decision making problems, however, are rarely straightforward as they are often influenced by incomplete and conflicting information. Thus, identifying the best possible decision after carefully diagnosing the particular issues has become

a critical challenge. Classical mathematical and statistical models frequently fail to capture the inherent complexity of such problems. This limitation has motivated researchers to develop advanced uncertainty handling framework that could better present real world decision environments. To address these challenges, the proposed structure based on advancement in neutrosophic soft set like models plays an important role in reducing uncertainty while enhancing the robustness of decision making models particularly for selecting most suitable object among several alternatives. The novelty of this structure lies in its hybrid combination because it combines the strengths of three powerful set theories including soft set, rough set and neutrosophic sets within a complex domain. Each domain contributes distinct computational advantages and their integration results in a more comprehensive and flexible mathematical framework capable of handling diverse forms of uncertainty. Soft set theory provides a flexible parameterized structure for capturing uncertainty without predefined membership functions. This feature makes this theory more useful in situation where decision parameters are expressed qualitatively. In real life applications particularly in health care and pharmaceutical industries decision makers rely on expert opinions, clinical experiences and qualitative assessments rather than quantitative data. More importantly, soft set structures allow such information to be incorporated naturally into decision making process and in the evaluation of alternatives across multiple criterion without imposing unrealistic precision. On the other hand, rough set theory deals uncertainty arising from incomplete knowledge by approximations such as lower and upper approximations. Lower approximation deals with those elements that certainly belong to that particular set. This approximation is crucial for data classification when information is insufficient to draw sharp boundaries between classes. Rough sets help in decision making problems to distinguish between definite and uncertain alternatives in recognizing and managing ambiguity. Neutrosophic set theory characterized each element using three independent membership components such as truth, false and neutral. This tri-component structure enables the explicit representation of incomplete and contradictory information which is encountered in complex decision making environments. Neutrosophic sets for instance allow an alternative that simultaneously exhibit favorable attributes and unfavorable aspects to be modeled concurrently by offering a more realistic representation of knowledge. Moreover, the incorporation of complex numbers into this hybrid structure enhances further its modeling power by enabling the representation of periodic and dynamic behaviors. In many real world applications particularly in biomedical and pharmaceutical centers processes are not static but evolve over time and may exhibit phase dependent patterns. Complex membership functions allow model to capture both magnitude and phase information supporting multifaceted uncertainty in real world data. This extension is useful

for modeling temporal variation in drug efficacy, patient response cycles and biomedical instructions that cannot be described adequately using real valued approaches. As a result, this comprehensive integration provides more informed and balanced decision even when data is incomplete and multidimensional. Therefore, the proposed framework enhances transparency and robustness. In addition, the application of proposed framework in pharmaceutical industry highlights its practical relevance. Pharmaceutical decision making processes are inherently complex and uncertain. There are some factors such as efficacy, safety, patient variability and

market consideration must all be evaluated based on incomplete evidence. By applying the proposed hybrid model, decision makers in pharmaceutical industry can systematically evaluate alternatives under uncertain conditions. This model supports more informed and resilient decision making by considering technical performance matrices and broader qualitative and subjective factors. This is important in contradictory situations where expert opinions may disagree. In nutshell, the proposed model in a complex domain offers a flexible approach for selecting the most suitable object in decision making problems under uncertain conditions. By integrating complementary strengths of soft, rough and neutrosophic sets in complex domain, the proposed model efficiently deals with roughness. On the basis of proposed mathematical framework an algorithm has been developed for giving more informed and balanced decisions for real world problems characterized by incomplete and contradictory information.

1.1. Literature review

The advancement of classical set theory began in 1965, Lofti-A Zadeh [9] coined the concept of Fuzzy theory. This allows elements to deal with a degree of belonging in an unit interval $[0, 1]$. Krassimir T. Atanassov [10] proposed Intuitionistic fuzzy set (IFS), the extension of fuzzy set concept that incorporate two degrees belonging and non belonging within the $[0, 1]$. In parallel to these developments, F Smarandache [12] introduced neutrosophic set (NS) by generalizing fuzzy concepts. This deals with three independent functions such as truth, indeterminacy and falsity in more flexible manner. Similarly, D Molodtsov [13] presented a soft set theory. This structure provides parameterized families of subsets. Building on this foundation, Cogman et al. [14] combined fuzzy set theory with soft sets and proposed fuzzy set to handle parameterized fuzzy data. Likewise, Maji [15] coined the Intuitionistic fuzzy soft set (IFSS) by integrating intuitionistic fuzzy theory into the soft set structure increasing its capability in parameterized contexts. Besides this, Picture Fuzzy soft set was introduced by Yang et al [16] by expanding intuitionistic fuzzy soft set. In addition, Maji [17] proposed Neutrosophic soft set (NSS) thereby enabling its ability to manage parameterized decision making scenarios. Further expansion followed, P Thirunavukarasu [18] proposed the complex fuzzy soft set (CFSS) capturing both magnitude and phase information. T. Kumar [19] presented complex intuitionistic fuzzy soft set (CIFSS) and broadening the representational capacity of soft set based models. The introduction of complex picture fuzzy soft set (CPFSS) by Akram et al [20] enables decision making in environments where uncertainty coexists in complex domains along with its membership functions. Additionally, F. Al Sharqi [21] combined neutrosophic

and soft concept in complex domain which is an advancement for highly complex systems. In parallel to these developments, Z Pawlak [22] proposed the Rough set theory (RST) which deals with vagueness by approximations such as upper and lower. Z.M Anwar [23] presented

intuitionistic fuzzy soft set with roughness for improvement in data classification. In same manner, R. Prasertpong [24] presented picture fuzzy soft set with roughness which improves decision making within complex systems. M.Das [25] and collaborators extended this structure by combining neutrosophic soft set with roughness to enhance the ability to deal with indeterminacy partial truth in uncertain environments.

1.2. *Research gap and Motivation*

Fuzzy set theory introduced uncertainty modeling but lacked explicit representation of hesitation between belonging and non-belonging. Intuitionistic and picture fuzzy sets improved this by incorporating non-belonging and neutrality. Soft sets deals with parameterization, enhancing flexibility, while their hybrid structure strengthened decision-making. Extensions of these frameworks into complex domains enabled handling periodic and multidimensional data. Neutrosophic sets further improved the field by modeling truth, indeterminacy, and falsity independently. Rough set theory serves boundary-based reasoning. The existing models fail to integrate parameterization, indeterminacy, and approximation within complex domain. Therefore, a unified hybrid model merging complex neutrosophic soft sets with rough approximations is imperative for more effective multidimensional uncertainty decision-making.

1.3. *Novelty of proposed research*

In this context, the proposed framework (CNsR) offers a powerful approach to handle uncertainty. By integrating complex-valued representation, parameterization, and boundary approximation, the model captures uncertainty more effectively. It represents membership through amplitude and phase components enabling the analysis of periodic and intricate data patterns. In addition, it incorporates three independent neutrosophic membership truth, indeterminacy, and falsity under parameter dependency providing a flexible and dynamic framework. Rough set theory enhances the model by distinguishing precise, boundary, and uncertain regions through lower and upper approximations. This combination further improves the accuracy and interpretability of decision-making processes. The framework is particularly valuable in health care applications where uncertainty and incomplete information are prevalent aiding in selecting optimal alternatives. Moreover, a decision-making algorithm has been developed to demonstrate

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its practical applicability. Therefore, this hybrid model establishes a robust foundation for next-generation intelligent systems, significantly enhancing precision and in complex decision-making environments characterized by dynamic and multidimensional uncertainty.

1.4. Arrangement of paper

This paper is systematically arranged to guide the readers to understand the basic concepts to advanced proposed hybrid mathematical framework along with its practical applications by applying an algorithm within this model. The arrangement of this paper is presented in the following sequence, section 1 refers to abstract and section 2 presents brief overview of existing studies and key limitations of existing models. This section also highlights motivation for developing a more adaptable hybrid model by integrating the strengths of existing models. Section 3 depicts the essential theoretical preliminaries and section 4 highlights characteristics of proposed structure CNsr. Similarly, section 5 establishes the application of proposed study along with algorithm and detailed case study for diagnosing a disease. Section 6 presents the comparison of proposed framework with existing studies and last section concludes the whole study briefly.

2. Preliminaries

Definition 2.1. Complex Intuitionistic fuzzy set: This set is denoted as C_{IF} over uni- versal set a is defined as $C_{IF} = \{(w, \mu_{C_{IF}}(w)), v_{C_{IF}}(w)) | w \in a\}$ where $\mu_{C_{IF}}(w), v_{C_{IF}}(w) :$

$a \rightarrow C$, $\mu_{C_{IF}}(w) = r_{C_{IF1}}(w)e^{j1\theta_{C_{IF1}}(w)}, v_{C_{IF}}(w) = r_{C_{IF2}}(w)e^{j2\theta_{C_{IF2}}(w)}$ here $\sqrt[r_{C_{IF1}}(w)]{r_{C_{IF1}}(w)} \in [0, 1], r_{C_{IF2}}(w) \in [0, 1]$ magnitude $\theta_{C_{IF1}}(w), \theta_{C_{IF2}}(w) \in [0, 2\pi)$ phase angle $j1, j2 = \sqrt{-1}$ is an imaginary unit.

Definition 2.2. Complex Picture fuzzy set: This set is denoted as CP_F over univer- sal set a is defined as $CP_F = \{(w, \mu_{CP_F}(w)), v_{CP_F}(w)) | w \in a\}$ where $\mu_{CP_F}(w), v_{CP_F}(w) :$

$a \rightarrow C$, $\mu_{CP_F}(w) = r_{CP_{F1}}(w)e^{j1\theta_{CP_{F1}}(w)}, v_{CP_F}(w) = r_{CP_{F2}}(w)e^{j2\theta_{CP_{F2}}(w)}$ here $\sqrt[r_{CP_{F1}}(w)]{r_{CP_{F1}}(w)} \in [0, 1], r_{CP_{F2}}(w) \in [0, 1]$ magnitude $\theta_{CP_{F1}}(w), \theta_{CP_{F2}}(w) \in [0, 2\pi)$ phase angle $j1, j2 = \sqrt{-1}$ is an imaginary unit.

Definition 2.3. Complex Neutrosophic set: [27] There lie complex valued membership functions in polar form instead of real numbers $[0, 1]$. A complex neutrosophic set CN in a is defined as: $CN = \{(w, \mu_{cn}(w), v_{cn}(w), \lambda_{cn}(w)) ; w \in a\}$ where every component can be expressed in polar form $\mu_{cn}(w) = r_{\mu}(w)e^{j\theta_{\mu}(w)}, v_{cn}(w) = r_v(w)e^{j\theta_v(w)}, \lambda_{cn}(w) = r_{\lambda}(w)e^{j\theta_{\lambda}(w)}$

Definition 2.4. Complex Fuzzy soft set: [28] It is denoted as CF_S . Let a is universal collection, M is parameters's set and $Q \subseteq M$ denotes relevant parameters's collection. A complex fuzzy soft set $CF_S = (Q, F)$ where $\mu : Q \rightarrow K(a)$ here $K(a)$ presents a collection of fuzzy subsets of a . For each parameter $m \in Q$.

$$CF_S = \{(w, \mu_{CF_S}(w)) | w \in a, \mu_{CF_S}(w) = r_{\mu_{CF_S}}(w)e^{j\theta_{\mu_{CF_S}}(w)}\}$$

Definition 2.5. Complex Intuitionistic Fuzzy soft set This set is denoted as CI_{FS} . Let a is universal set, M denotes family of parameters and $Q \subseteq M$ is a collection of relevant parameters for a particular situation. Set CI_{FS} over universal set a with respect to Q is defined as $CI_{FS} = \{\langle w, \mu_{CI_{FS}}(w), \nu_{CI_{FS}}(w) \rangle \mid w \in a\}$ where $\mu_{CI_{FS}}(w) = r_{\mu_{CI_{FS1}}}(w)e^{j\theta_{\mu_{CI_{FS1}}}(w)}$, $\nu_{CI_{FS}}(w) = r_{\nu_{CI_{FS2}}}(w)e^{j\theta_{\nu_{CI_{FS2}}}(w)}$ denote belonging , non belonging values respectively in CI_{FS} .

3. Complex neutrosophic soft set with roughness(CNsR)

This section defines the main mathematical structure CNsR and its example of this study.

Definition 3.1. Let Ω be a non-empty universe of discourse and let A be the family of attributes. Suppose $B \subseteq A$ and let $CNS(\Omega)$ denote the class of all complex neutrosophic sets on Ω . Consider a complex neutrosophic soft set

$$S = (\Psi_{CN}, B),$$

where the mapping

$$\Psi_{CN} : B \rightarrow CNS(\Omega)$$

assigns to each attribute in B a complex neutrosophic set over Ω . For any subset $Y \subseteq \Omega$, the CN_B -lower approximation of Y is defined by

$$\underline{CN}_B(Y) = \{(b, \{\langle x/\tau_B(x), \iota_B(x), \varphi_B(x) \rangle\})\},$$

where

$$\tau_B(x) = \rho_\tau(x)e^{i\omega_\tau(x)}, \quad \iota_B(x) = \rho_\iota(x)e^{i\omega_\iota(x)}, \quad \varphi_B(x) = \rho_\varphi(x)e^{i\omega_\varphi(x)}.$$

The corresponding CN_B -upper approximation of Y is given by

$$\overline{CN}_B(Y) = CN_B(\Omega) - \underline{CN}_B(Y^c).$$

Example 3.2. Let

$$\Omega = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$$

be the universe set and

$$A = \{a_1, a_2, a_3, a_4\}$$

be the collection of attributes. Define a mapping

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$$\Psi : A \rightarrow P(\Omega)$$

by

$$\Psi(a_1) = \{x_1, x_3, x_5\}, \quad \Psi(a_2) = \emptyset,$$

$$\Psi(a_3) = \{x_3, x_4\}, \quad \Psi(a_4) = \{x_2, x_6\}.$$

Now define the complex neutrosophic soft mapping

$$\Psi_{CN} : A \rightarrow CNS(\Omega)$$

as follows:

$$\Psi_{CN}(a_1) = \left(\frac{x_1}{0.6e^{i2\pi(0.3)}, 0.4e^{i2\pi(0.2)}, 0.2e^{i2\pi(0.1)}} , \frac{x_2}{0.0e^{i2\pi(0.2)}, 0.7e^{i2\pi(0.3)}, 1.0e^{i2\pi(0.2)}} , \right. \\ \left. \frac{x_3}{0.8e^{i2\pi(0.4)}, 0.3e^{i2\pi(0.1)}, 0.2e^{i2\pi(0.3)}} , \frac{x_4}{0.0e^{i2\pi(0.1)}, 0.6e^{i2\pi(0.2)}, 1.0e^{i2\pi(0.4)}} , \right. \\ \left. \frac{x_5}{0.7e^{i2\pi(0.5)}, 0.5e^{i2\pi(0.2)}, 0.1e^{i2\pi(0.3)}} , \frac{x_6}{0.0e^{i2\pi(0.3)}, 0.8e^{i2\pi(0.1)}, 1.0e^{i2\pi(0.2)}} , \right. \\ \left. \frac{x_7}{0.0e^{i2\pi(0.4)}, 0.9e^{i2\pi(0.3)}, 1.0e^{i2\pi(0.1)}} , \frac{x_8}{0.0e^{i2\pi(0.2)}, 0.6e^{i2\pi(0.4)}, 1.0e^{i2\pi(0.2)}} \right).$$

Here, only x_1, x_3, x_5 are effectively selected in $\Psi_{CN}(a_1)$ because the remaining elements have zero membership and full non-membership. Similarly, let

$$\Psi_{CN}(a_2) = \emptyset, \\ \Psi_{CN}(a_3) = \left(\frac{x_3}{0.8e^{i2\pi(0.4)}, 0.3e^{i2\pi(0.1)}, 0.2e^{i2\pi(0.3)}} , \frac{x_4}{0.6e^{i2\pi(0.2)}, 0.4e^{i2\pi(0.3)}, 0.3e^{i2\pi(0.1)}} , \right. \\ \text{and}$$

$$\Psi_{CN}(a_4) = \left(\frac{x_2}{0.7e^{i2\pi(0.3)}, 0.5e^{i2\pi(0.2)}, 0.2e^{i2\pi(0.4)}} , \frac{x_6}{0.9e^{i2\pi(0.1)}, 0.2e^{i2\pi(0.3)}, 0.1e^{i2\pi(0.2)}} \right).$$

Therefore, the complex neutrosophic soft set over Ω is

$$(\Psi_{CN}, A) = \{(a_1, \Psi_{CN}(a_1)), (a_2, \Psi_{CN}(a_2)), (a_3, \Psi_{CN}(a_3)), (a_4, \Psi_{CN}(a_4))\}.$$

Now take

$$Y = \{x_3, x_4, x_6\}, \quad Y^c = \{x_1, x_2, x_5, x_7, x_8\} \subseteq \Omega,$$

and let

$$B = \{a_1, a_3, a_4\} \subseteq A.$$

Then the restricted complex neutrosophic soft set is

$$(\Psi_{CN}, B) = \{(a_1, \Psi_{CN}(a_1)), (a_3, \Psi_{CN}(a_3)), (a_4, \Psi_{CN}(a_4))\} = CN_B(\Omega).$$

Hence, the CN_B -lower approximation of Y is

$$\underline{apr} CN(Y) = (a, \{ \frac{x_3}{0.8e^{i2\pi(0.4)}, 0.3e^{i2\pi(0.1)}, 0.2e^{i2\pi(0.3)}} , \frac{x_4}{0.6e^{i2\pi(0.2)}, 0.4e^{i2\pi(0.3)}, 0.3e^{i2\pi(0.1)}} \}) .$$

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^{3}

$0.8e^{i2\pi(0.4)}, 0.3e^{i2\pi(0.1)}, 0.2e^{i2\pi(0.3)}$

$0.6e^{i2\pi(0.2)}, 0.4e^{i2\pi(0.3)}, 0.3e^{i2\pi(0.1)}$

Also,

$$\underline{apr} \, CN \, (Y^c) = (a, \{ \text{-----} x_2 \text{-----}, \text{-----} x_6 \text{-----} \}) .$$

$$\begin{array}{c}
\overline{B} \quad \quad \quad 4 \quad \quad \quad 0.7e^{i2\pi(0.3)}, 0.5e^{i2\pi(0.2)}, 0.2e^{i2\pi(0.4)} \quad \quad \quad 0.9e^{i2\pi(0.1)}, 0.2e^{i2\pi(0.3)}, 0.1e^{i2\pi(0.2)} \\
\hline
\text{Therefore, the } CN_B\text{-upper approximation of } Y \text{ is} \\
\text{apr } CN_B(Y) = (a_1, \{ \frac{x_1}{0.6e^{i2\pi(0.3)}, 0.4e^{i2\pi(0.2)}, 0.2e^{i2\pi(0.1)}} , \frac{x_3}{0.8e^{i2\pi(0.4)}, 0.3e^{i2\pi(0.1)}, 0.2e^{i2\pi(0.3)}} , \\
\frac{x_5}{0.7e^{i2\pi(0.5)}, 0.5e^{i2\pi(0.2)}, 0.1e^{i2\pi(0.3)}} \}), (a_3, \{ \frac{x_3}{0.8e^{i2\pi(0.4)}, 0.3e^{i2\pi(0.1)}, 0.2e^{i2\pi(0.3)}} ,
\end{array}$$

$$\frac{x_4}{0.6e^{i2\pi(0.2)}, 0.4e^{i2\pi(0.3)}, 0.3e^{i2\pi(0.1)}} \} \} .$$

Thus, Y is a rough set with respect to the given information system CN_B , since

$$\underline{apr} CN_B(Y) \neq \overline{apr} CN_B(Y).$$

Otherwise, Y is called a CN_B -definable set.

3.1. Problem statement and general discussion

In contemporary research, the advancement in artificial intelligence (AI) and machine learning aims to improve the quality of human life across the multiple disciplines. In this regard, the use of AI in decision support systems in disease diagnosing has gained considerable importance. Medical specialists face challenges when most suitable object has to be selected in a given amount of information. To address these problems, the application of a proposed mathematical structure CNsR provides an advance way to handle uncertainty in multi attribute problems of decision making. To handle these issues an advanced algorithm has been developed based on the proposed mathematical framework for medical assistance. This algorithmic approach utilises the strengths of lower and upper approximation operators to define objects thereby improving the reliability of recognition under incomplete information. Optimistically, this improves decision accuracy and also makes the process computationally effective. Moreover, this framework is highly important for a wide range of multi-attribute decision making (MAMD) scenarios.

3.2. Algorithm

The proposed algorithm based on CNsR structure consist of steps are described as follows.

- Step 1:** Consider a set a as a universal set, whereas D denotes the set of parameters or attributes and α be a CNs relation from set a to set of parameters D .
- Step 2:** Set S is an complex neutrosophic soft set on the set of parameters D is supposed to be a set of most suitable object.
- Step 3:** Define lower and upper approximations denoted by $\alpha(S)$ and $\underline{\alpha}(S)$ of set S with respect to (a, D, α) .
- Step 4:** Applying upper and lower approximations on a required given set.
- Step 5:** Find direct sum between two CNsR, which are defined as $\alpha(S)$ and $\underline{\alpha}(S)$.
- Step 6:** Calculate cosine similarity measure of $\alpha(S)$ and $\underline{\alpha}(S)$ in complex form that is $CS(n_w, n_{w_i})$.
- Step 7:** In this step, now find out the results of direct sum set by using above cosine similarity measure formula.
- Step 8:** Arrange similarity measure from lowest to the highest.
- Step 9:** Choose the alternative with highest similarity measure.

3.3. Case study

An algorithm has been designed to diagnose one disease among three, namely Covid-19, Influenza, and Pneumonia, as a case study on the basis of their symptoms. These diseases have some common symptoms, for instance, fever, cough, fatigue, and shortness of breath. By making use of this information, a universal set containing symptoms is defined as

$$\{w_1 = \text{fever}, w_2 = \text{cough}, w_3 = \text{fatigue}, w_4 = \text{shortness of breath}\},$$

and the set of parameters containing three different diseases, Covid-19 (d_C), Influenza (d_I), and Pneumonia (d_P), is defined as

$$\{d_C, d_I, d_P\}.$$

This information is presented according to the proposed CNsR structure for more precise decision-making. Using these two sets, a framework is constructed for diagnosing one disease among three diseases based on their common symptoms.

Step 1: Using the universal set $\alpha = \{w_1, w_2, w_3, w_4\}$ and the set of parameters $D = \{d_C, d_I, d_P\}$, the constructed CNsR structure for three diseases Covid-19 (d_C), Influenza (d_I), and Pneumonia (d_P) is:

α	d_C	d_I	d_P
w_1	$0.3e^{i2\pi 0.2}, 0.5e^{i2\pi 0.9}, 0.8e^{i2\pi 0.2}$	$0.5e^{i2\pi 0.2}, 0.4e^{i2\pi 0.1}, 0.6e^{i2\pi 0.1}$	$0.2e^{i2\pi 0.3}, 0.3e^{i2\pi 0.7}, 0.6e^{i2\pi 0.5}$
w_2	$0.7e^{i2\pi 0.3}, 0.4e^{i2\pi 0.2}, 0.2e^{i2\pi 0.8}$	$0.6e^{i2\pi 0.3}, 0.5e^{i2\pi 0.2}, 0.4e^{i2\pi 0.1}$	$0.9e^{i2\pi 0.1}, 0.4e^{i2\pi 0.2}, 0.8e^{i2\pi 0.4}$
w_3	$0.5e^{i2\pi 0.4}, 0.4e^{i2\pi 0.3}, 0.2e^{i2\pi 0.5}$	$0.6e^{i2\pi 0.3}, 0.5e^{i2\pi 0.2}, 0.3e^{i2\pi 0.1}$	$0.1e^{i2\pi 0.9}, 0.2e^{i2\pi 0.5}, 0.4e^{i2\pi 0.8}$
w_4	$0.9e^{i2\pi 0.1}, 0.7e^{i2\pi 0.2}, 0.1e^{i2\pi 0.2}$	$0.7e^{i2\pi 0.1}, 0.5e^{i2\pi 0.3}, 0.2e^{i2\pi 0.4}$	$0.3e^{i2\pi 0.1}, 0.2e^{i2\pi 0.8}, 0.7e^{i2\pi 0.1}$

Step 2: Now, suppose

$$S = \langle d_C, 0.2e^{i2\pi 0.4}, 0.5e^{i2\pi 0.1}, 0.3e^{i2\pi 0.9} \rangle, \langle d_I, 0.3e^{i2\pi 0.9}, 0.8e^{i2\pi 0.1}, 0.2e^{i2\pi 0.3} \rangle, \langle d_P, 0.5e^{i2\pi 0.1}, 0.2e^{i2\pi 0.9}, 0.1e^{i2\pi 0.2} \rangle$$

is the most suitable object that is a complex neutrosophic soft set on the set of parameters.

Step 3: In this step, define the upper complex neutrosophic soft approximation (UCNs) and the lower complex neutrosophic soft approximation (LCNs) of S with respect to the universal set and the set of parameters as

$$\bar{\alpha}(S) = \{ w, T_{\bar{\alpha}(S)}(w), I_{\bar{\alpha}(S)}^-(w), F_{\bar{\alpha}(S)}^-(w) ; w \in a \}$$

and

$$\underline{\alpha}(S) = \{ w, T_{\underline{\alpha}(S)}(w), I_{\underline{\alpha}(S)}(w), F_{\underline{\alpha}(S)}(w) ; w \in a \},$$

where

$$T_{\bar{\alpha}(S)}(w) = \max T_{\alpha(S)}(w, d), \min T_S(w) ,$$

$$I_{\bar{\alpha}(S)}(w) = \min I_{\alpha(S)}(w, d), \max T_S(w) ,$$

$$F_{\bar{\alpha}(S)}(w) = \min F_{\alpha(S)}(w, d), \max F_S(w) ,$$

$$T_{\underline{\alpha}(S)}(w) = \min F_{\alpha(S)}(w, d), \max T_S(w) ,$$

$$I_{\underline{\alpha}(S)}(w) = \max 1 - I_{\alpha(S)}(w, d), \min I_S(w) ,$$

$$F_{\underline{\alpha}(S)}(w) = \max T_{\alpha(S)}(w, d), \min F_S(d) .$$

Step 4: By applying upper and lower approximations, our required sets have been calculated as follows. Therefore, $(\bar{\alpha}(S), \underline{\alpha}(S))$ is a CNsT set in the complex neutrosophic soft approximation space (a, D, α) , where

$$\begin{aligned} \bar{\alpha}(S) &= \{ (w_1, 0.3e^{i2\pi(0.9)}, 0.3e^{i2\pi(0.7)}, 0.6e^{i2\pi(0.1)}), \\ &\quad (w_2, 0.5e^{i2\pi(0.1)}, 0.4e^{i2\pi(0.2)}, 0.3e^{i2\pi(0.9)}), \\ &\quad (w_3, 0.3e^{i2\pi(0.9)}, 0.2e^{i2\pi(0.9)}, 0.3e^{i2\pi(0.1)}), \\ &\quad (w_4, 0.3e^{i2\pi(0.9)}, 0.2e^{i2\pi(0.9)}, 0.2e^{i2\pi(0.4)}) \} \\ \underline{\alpha}(S) &= \{ (w_1, 0.6e^{i2\pi(0.1)}, 0.8e^{i2\pi(0.02)}, 0.3e^{i2\pi(0.2)}), \\ &\quad (w_2, 0.2e^{i2\pi(0.8)}, 0.9e^{i(-0.06)}, 0.3e^{i2\pi(0.9)}), \\ &\quad (w_3, 0.2e^{i2\pi(0.5)}, 1.2e^{i(0)}, 0.3e^{i2\pi(0.9)}), \\ &\quad (w_4, 0.2e^{i2\pi(0.4)}, 1.2e^{i(0.9)}, 0.3e^{i2\pi(0.9)}) \} \end{aligned}$$

Step 5: Now, the sum of both lower CNsR approximation and upper CNsR approximation operators $\bar{\alpha}(S)$ and $\underline{\alpha}(S)$ is again a CNsR collection, that is,

$$\alpha(S) \oplus \alpha(S) = \begin{pmatrix} (w_1, 0.2e^{i0.1}, 0.2e^{i0.7}, 0.1e^{i0.3}), \\ (w_2, 0.4e^{i0.06}, 0.3e^{i0.1}, 0.09e^{i0.8}), \\ (w_3, 0.2e^{i0.8}, 0.2e^{i0.9}, 0.09e^{i0}), \\ (w_4, 0.2e^{i0.5}, 0.2e^{i0.8}, 0.06e^{i0.31}) \end{pmatrix}$$

Step 6: Put the cosine similarity measure formula in complex form:

$$CS(n_w, n_{w_i}) = |r_{j1} + a_{i1}| \times |2\pi\theta_{j1} + 2\pi b_{i1}| + |r_{j3} + a_{i3}| \times |2\pi\theta_{j3} + 2\pi b_{i3}| - |r_{j2} + a_{i2}| \times |2\pi\theta_{i2} + 2\pi b_{i2}|$$

Step 7: By using this formula, the following results are calculated:

$$CS(w_1, w_2) = -1.6779, \quad CS(w_1, w_3) = -2.0998, \quad CS(w_1, w_4) = -1.871, \quad CS(w_2, w_3) = 0.27,$$

$$CS(w_2, w_4) = -0.321, \quad CS(w_3, w_4) = 0.75$$

Step 8: Arrange the calculated results from least to highest for choosing the most appropriate object:

$$CS(w_1, w_3) < CS(w_1, w_4) < CS(w_1, w_2) < CS(w_2, w_4) < CS(w_2, w_3) < CS(w_3, w_4)$$

Step 9: Therefore, choose the highest alternative with similarity measure as the best suitable outcome.

4. Comparison

In this section comparison is made, comparison of proposed model (the advancement of complex neutrosophic soft set like models) with existing structures like neutrosophic soft set [17], complex neutrosophic soft set [21], neutrosophic soft set with roughness [25], picture fuzzy soft set [32], complex picture fuzzy soft set [20], picture fuzzy soft set with roughness [24] and complex picture fuzzy soft set with roughness. In this regard, first of all the comparison of proposed structure with neutrosophic soft set is presented in which three membership degrees truth such as indeterminacy and falsity deal with static uncertainty under parameterized conditions in a unit closed interval. Whereas, the proposed structure deals with imprecise data in decision making for analysing complex and phase dependent data also captures dynamic uncertainty in systems where oscillating behaviour is observed. Further, complex neutrosophic soft set deals uncertainty by assigning membership degrees, useful for dynamic systems because it treats all elements equally without differentiating between certain and uncertain boundaries.

4.1. Numerical Comparison with baseline models

For examination of practical effectiveness of the proposed structure, a numerical comparison is carried out. In comparison, simpler baseline models are used on the same case study, which deals with disease diagnose. The purpose of numerical comparison is to show the strength of proposed model (CNSR), in contrast of reduced model which use limited information. An amplitude based single valued neutrosophic soft set baseline is considered for this purpose. In this method, only the amplitude values of complex neutrosophic values are used while ignoring phase terms and rough approximations. Suppose

$$S = \{(d_C, 0.2, 0.5, 0.3), (d_I, 0.3, 0.8, 0.2), (d_P, 0.5, 0.2, 0.1)\}$$

be the desired set. For each alternative w_i , the Hamming distance from S is defined as:

$$d_H(w_i, S) = \sum_{j=1}^m (|T_{ij} - T_{Sj}| + |I_{ij} - I_{Sj}| + |F_{ij} - F_{Sj}|)$$

set $m = 3$ is the number of parameters and corresponding normalized score is given as

$$S_H(w_i) = 1 - \frac{d_H(w_i, S)}{3m}$$

In the same manner, Euclidean baseline distance is defined as:

$$d_E(w_i, S) = \sqrt{\sum_{j=1}^m [(T_{ij} - T_{Sj})^2 + (I_{ij} - I_{Sj})^2 + (F_{ij} - F_{Sj})^2]}$$

and corresponding score is

$$S_E(w_i) = \frac{1}{1 + d_E(w_i, S)}$$

By using amplitude values taking from case study the results are calculated.

$$d_H(w_1, S) = 2.4, d_H(w_2, S) = 2.8, d_H(w_3, S) = 1.9, d_H(w_4, S) = 2.6$$

which yields

$$S_H(w_1) = 0.733, S_H(w_2) = 0.688, S_H(w_3) = 0.781, S_H(w_4) = 0.711$$

Therefore, Hamming baseline ranking is

$$w_3 > w_1 > w_4 > w_2$$

Similarly, the Euclidean baseline gives

$$d_E(w_1, S) = 0.98, d_E(w_2, S) = 1.086, d_E(w_3, S) = 0.741, d_E(w_4, S) = 0.787$$

and corresponding scores are

$$S_E(w_1) = 0.50, S_E(w_2) = 0.47, S_E(w_3) = 0.57, S_E(w_4) = 0.55$$

Hence, the Euclidean baseline ranking is

$$w_3 > w_4 > w_1 > w_2$$

5. Conclusion

The discussion presented in this study leads to the conclusion that the proposed CNsR structure offers a significant contribution to decision-making methodologies. This hybrid structure integrates neutrosophic sets, soft set, and rough set theories within a complex domain resulting in a more flexible and adaptive framework for handling uncertainty. This approach is particularly effective in addressing indeterminate and inconsistent information that frequently arises in real-world decision-making scenarios. By incorporating three distinct membership degrees the model enables a more detailed and comprehensive evaluation of uncertain parameters. Comparative analysis with existing models shows that the proposed framework exhibits superior robustness, flexibility, and precision. It also highlights key differences between traditional approaches and the proposed structure. The developed decision-making algorithm further enhances the model by efficiently identifying optimal attributes. This contributes to improved decision outcomes, particularly in ambiguous environments. In nutshell, the proposed hybrid model successfully overcomes the shortcomings of conventional techniques and establishes a powerful computational framework for intelligent decision support systems.

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