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Computational Modeling Of Heat Transfer In A 2d Square Plate Using Finite Difference Scheme

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ABSTRACT

The finite difference technique to solve the 2D heat equation and determine how heat may be diffused within a square plate according to certain boundary constraints. The user can enter his own material's thermal conductivity factor, specific heat, and density into the simple MATLAB code, and the program will solve for a wide variety of materials, including common ones like copper, aluminum, silver, and others. In addition, the user-defined boundary conditions, known as "Dirichlet Conditions," are also supported. Time required for the plate to attain a steady state temperature is also calculated, with an error margin chosen in advance by the user. Selecting more nodes in the x and y axes before meshing and beginning iterative solution searching can improve accuracy. The algorithm provides two methods for solving the time derivative (Euler and 2nd order Runge-Kutta) and a central finite difference method for discretizing the space derivative. After the problem has been solved, a graphical simulation in time appearance reveals how heat spreads over the plate in the allotted time.

Keywords: Heat Equation; 2D Square Plate; Finite Difference Technique; Runge-Kutta Scheme

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Introduction

The partial differential equation can be defined as an equation whose derivatives include an unknown function for two or more independent variables, and partial differential equations can describe many phenomena, including wave propagation, fluid flow (air or liquid), vibration, solid mechanics, flow and distribution of heat, fields, electric potentials, and the diffusion of chemicals. In air or water, electromagnetism and quantum mechanics [1– 3].

The study of heat transfer is of significant importance in various fields of engineering and science. Understanding the behavior of heat flow is crucial for designing efficient thermal systems, predicting temperature distributions, and optimizing energy consumption. The heat equation, a fundamental partial differential equation, describes the distribution of heat in a given medium over time. Solving the heat equation allows us to model and analyze heat transfer phenomena accurately.

The finite difference method (FDM) is a widely used numerical technique for solving partial differential equations, including the heat equation. It discretizes the continuous domain into a set of discrete points, approximating the derivatives in the governing equation using different formulas. By converting the continuous problem into a discrete one, FDM enables us to solve complex heat transfer problems on a computer with reasonable accuracy and computational efficiency.

One of the most interesting features of solving partial differential equations using finite difference methods is that many methods are based on linear equations with constant coefficients, it is a partial derivative. Thus, finite differences methods can be used, which are classified into three categories: the explicit, implicit, and semi-implicit methods (Crank-Nicolson), and most of the methods have defects in stability and accuracy [4–6].

The first who invented the Crank-Nicolson method were the two scientists John Crank and Phyllis Nicolson in the 1940s, who originally applied it to the heat equation and approached the solution of the heat equation to some finite networks by approximating the derivatives in space x and time t with finite differences, and much earlier, The Richardson devised the Finite Difference Chart which was easy to calculate but was numerically unstable and was not useful [7]. It was not until John Crank, Nicolson Phyllis, and others performed extended numerical computations that the instability of the difference graph was recognized. Therefore, the Crank-Nicolson method is numerically stable and requires only solving a very simple system of linear equations at each time level. Thus, the Crank-Nicolson method has become one of the most common methods [4], [5], [8], [9]. One of the downsides to the Crank-Nicholson method is that it produces a complex system of linear equations despite the availability of approximate solutions efficiency.

The implicit method is characterized by being unconditionally stable, the time variable is precisely controlled, and it can extract the value of the time variable with fewer steps. In any case, it is considered one of the most difficult methods when calculating two- dimensional and three-dimensional. It has the same number of steps as the explicit method in the first dimension [5], [10].

In this research, we discuss the solution of complex problems in ordinary differential equations and partial differential equations using a simple method called finite difference. A finite difference is a mathematical expression of the form f (x + b) f (x +

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a). If a finite difference is divided by b − a, one gets a difference quotient. The approximation of derivatives by finite differences plays a central role in finite difference methods for the numerical solution of differential equations, especially boundary value problems. In this report, we will use the central finite difference in space and forward finite difference in time to solve the heat equation in a 2D- Plate with some fixed initial conditions called "Dirichlet Conditions" to obtain a solution that describes the temperature of each node in space in any time within the interval entered before debugging. We will also find out more about the steady state of heat diffusion throughout the plate which is defined as the state when the system reaches it, the solution to the heat equation will be independent on time. This can be interpreted physically as the change in temperature with time will become insignificant, can be neglected and the truncation error here will be defined as a tolerance.

The Numerical solution of PDEs is a rich and active field of modern applied mathematics. The steady growth of the subject is stimulated by ever-increasing demands from the natural sciences, engineering and economics to provide accurate and reliable approximations to mathematical models involving partial differential equations (PDEs) whose exact solutions are either too complicated to determine in closed form or, in many cases, are not known to exist. While the history of numerical solutions of ordinary differential equations is firmly rooted in 18th and 19th century mathematics, the mathematical foundations of the field of numerical solution of PDEs are much more recent: they were first formulated in the landmark paper “Uber die partiellen Differenzengleichungen der mathematis” chen Physik (On the partial difference equations of mathematical physics) by Richard Courant, Karl Friedrichs, and Hans Lewy, published in 1928. There is a vast array of powerful numerical techniques for specific PDEs, level set and fast marching methods for front-tracking and interface problems, numerical methods for PDEs on, possibly evolving, manifolds, immersed boundary methods, mesh-free methods, particle methods, vortex methods, various numerical homogenization methods and specialized numerical techniques for multiscale problems, wavelet-based multiresolution methods, sparse finite difference or finite element methods, greedy algorithms and tensorial methods for high-dimensional PDEs, domain-decomposition methods for geometrically complex problems, and numerical methods for PDEs with stochastic coefficients that feature in a number of applications, including uncertainty quantification problems.

The objective of this research is to solve the two-dimensional heat equation using the finite difference method. The heat equation in two dimensions is a partial differential equation that governs the distribution of temperature over a two-dimensional domain. By discretizing the domain using finite difference approximations, we aim to obtain a numerical solution that accurately predicts the temperature distribution at different points in the domain at different time steps.

The scope of this manuscript is to develop a comprehensive understanding of the finite difference method and its application to solving the two-dimensional heat equation. The specific objectives of this study are as follows:

To derive the finite difference approximation for the two-dimensional heat equation and develop the discrete system of equations

To implement a numerical algorithm based on the derived finite difference

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approximation and develop a computational code for solving the two-dimensional heat equation.
To validate the numerical results obtained from the finite difference method by comparing them with analytical solutions or experimental data.
To investigate the convergence and stability of the numerical scheme and analyze the computational efficiency of the implemented algorithm.
The study of the finite difference method for solving the two-dimensional heat equation has significant implications in various engineering disciplines. Some of the potential applications include thermal analysis of electronic components, heat transfer in materials processing, geothermal systems, and environmental modeling. By accurately predicting temperature distributions in such systems, engineers and scientists can optimize their designs, enhance energy efficiency, and improve overall performance.
Furthermore, this research contributes to the existing body of knowledge by providing a detailed investigation of the finite difference method for solving the two-dimensional heat equation. It will serve as a valuable resource for researchers, engineers, and students seeking to understand and apply numerical techniques for heat transfer analysis.

Mathematical Modelling
In this section we recall the basic notion of mathematical modelling for the Partial Differential Equations. Some characterizations are also investigated.

Modeling of Two – Dimensional Heat Equation
This differential equation governs the temperature distribution (in the transient state) in a thin metal plate of uniform heat conducting (i.e., homogeneous) material. Let a metal plate be of uniform thickness h and let the plate be represented by the $xy-$ plane. The lateral (top and bottom) faces of the plate are covered by layers of thermally insulating material so that we may neglect any heat flow in the directions perpendicular to its faces. Thus, heat can enter or leave the plate only through its edges. Consequently, the heat flow is mainly in the x and y directions and the temperature function u at any point of the plate depends only on x and y coordinates and the time t i.e., $u = u(x, y, t)$
We make the following assumptions,
The plate is sufficiently thin so that the heat flow in it can be regarded as two – dimensional.
The plate is of uniform thickness and homogeneous material, therefore its thermal conductivity K, the specific heat σ, and the density ρ (mass per unit area) are constants.
Consider a small rectangular element $PQRS$ of the plate with sides $\Delta x \; and \; \Delta y$ as shown in Figure 2.1.

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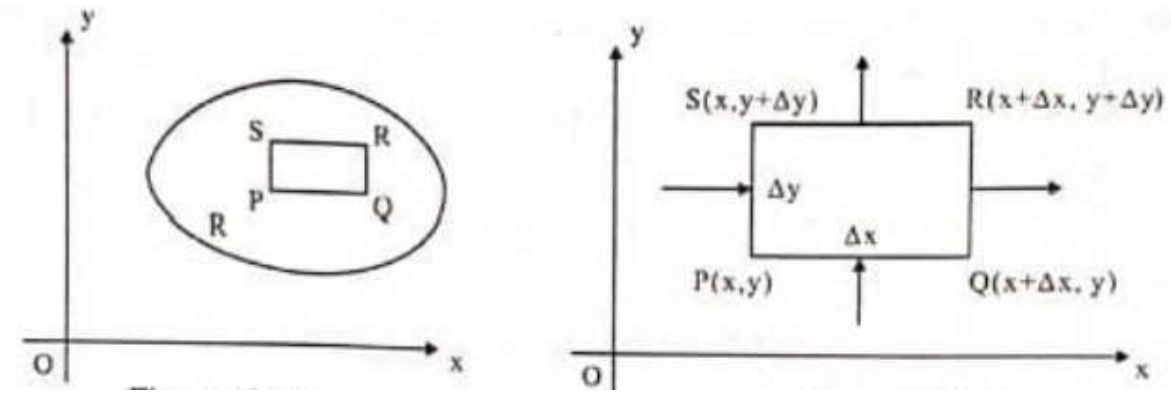

Figure 2.1: A rectangular element in the xy-plane

Since the area of the edge PS is h Δy, the amount of heat entering the element through the edge PS in time $\Delta t = -K\,(\text{h}\,\Delta y)\left(\frac{\partial u}{\partial x}\right)_x \Delta t$

and the amount of heat leaving the element through the edge QR in time Δt

$= -K\,(\text{h}\,\Delta y)\left(\frac{\partial u}{\partial x}\right)_{x+\Delta x} \Delta t$

Also, the amount of heat entering the element through the PQ in time Δt

$= -K\,(\text{h}\,\Delta x)\left(\frac{\partial u}{\partial y}\right)_y \Delta\text{t}$

And the amount of heat leaving the element through the edge SR in time Δt

$= -K\,(\text{h}\,\Delta x)\left(\frac{\partial u}{\partial y}\right)_{y+\Delta y} \Delta\text{t}$

Hence the amount of heat retained by the element PQRS in time Δt is given by the amount of heat entering minus the amount of heat leaving

$= -K\,(\text{h}\,\Delta y)\left(\frac{\partial u}{\partial x}\right)_x \Delta t - K\,(\text{h}\,\Delta x)\left(\frac{\partial u}{\partial y}\right)_y \Delta\text{t}$

$+ K\,(\text{h}\,\Delta y)\left(\frac{\partial u}{\partial x}\right)_{x+\Delta x} \Delta t + K\,(\text{h}\,\Delta x)\left(\frac{\partial u}{\partial y}\right)_{y+\Delta y} \Delta\text{t}$

$= \text{K}\left[\left(\frac{\partial u}{\partial x}\right)_{x+\Delta x} - \left(\frac{\partial u}{\partial x}\right)_x\right] \text{h}\,\Delta y \Delta x + K\left[\left(\frac{\partial u}{\partial y}\right)_{y+\Delta y} - \left(\frac{\partial u}{\partial y}\right)_y\right] h\,\Delta x\,\Delta t$

This amount of heat raises the temperature of the element $PQRS$ by an amount Δu.

Also, we know that if ρ, is the density of the plate and σ specifies heat, then the amount of heat required to raise the temperature of the element $PQRS$ by Δu

$= \rho(h\Delta x\,\Delta y)\,\sigma\,\Delta u$

3 Finite Difference Method

In this section we will define Finite Difference Method and will also discuss its properties and examples.

Difference Quotients

A difference quotient is the quotient obtained by dividing the difference between two values of a function by the difference between the two corresponding values of the independent variable. Thus, for a function $u(x)$ of a single variable, the difference coefficient is,

$$\frac{u(x+h)-u(x)}{h}$$

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Whose limiting value is the derivative of $u(x)$ with respect to x, that is $\frac{du}{dx}$.

Here $u(x,y)$ is a function of two independent variable x, y. Therefore, we have considered the differences in both.

First let us consider the differences in the x-direction. Taylor's series for $u(x,y_0)$ about the point (x_0,y_0) is,

$$u(x,y_0) = u(x_0,y_0) + (x-x_0) + u_x(x_0,y_0) + \frac{(x-x_0)^2}{2!}u_{xx}(\xi,y_0)$$

Where $x_0 \le \xi \le x$. Let $x = x_0 + h$ in equation.

$$u(x_0+h,y_0) = u(x_0,y_0) + hu_x(x_0,y_0) + \frac{h^2}{2!}u_{xx}(\xi,y_0).$$

Or,

$$\frac{u(x_0+h,y_0)-u(x_0,y_0)}{h} = u_x(x_0,y_0) + \frac{h^2}{2!}u_{xx}(\xi,y_0)$$

$$u_x(x_0,y_0) = \frac{u(x_0+h,y_0)-u(x_0,y_0)}{h} - \frac{h}{2!}u_{xx}(\xi,y_0).$$

Where $x_0 \le \xi \le x_0 + h$.

Equation implies that, if we replace $u_x(x_0,y_0)$ by,

$$\frac{u(x_0+h,y_0)-u(x_0,y_0)}{h}$$

The truncation error = $-\frac{h}{2!}u_{xx}(\xi,y_0)$; $x_0 \le \xi \le x_0 + h$,

Hence, we have the finite difference formula for the first partial derivative at (x_0,y_0) as,

$$u_x(x_0,y_0) = \frac{u(x_0+h,y_0)-u(x_0,y_0)}{h}.$$

Which is called forward difference approximation to $u_x(x_0,y_0)$.

Similarly, by putting $x = x_0 - h$ in equation or by changing h to$-h$ in equation we get

$$u_x(x_0,y_0) = \frac{u(x_0,y_0)-u(x_0=h,y_0)}{h}.$$

Which is known as backward difference approximation to $u_x(x_0,y_0)$.

Now to obtain an approximation for u_{xx}, we use both forward and backward differences.

Using forward difference for u_x, we get

$$u_{xx}(x_0,y_0) = \frac{u(x_0+h,y_0)-u(x_0,y_0)}{h}.$$

Since there is a bias in forward direction, we use backward differences for u_x to avoid this effect, i.e.

$$u_{xx}(x_0,y_0) = \frac{u(x_0,y_0)-u(x_0-h,y_0)}{h}.$$

Changing x_0 to $x_0 + h$ in equation we get,

$$u_x(x_0+h,y_0) = \frac{u(x_0+h,y_0)-u(x_0,y_0)}{h}.$$

Substituting equation

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$$u_{xx}(x_0, y_0) = \frac{u(x_0 + h, y_0) - 2u(x_0, y_0) + u(x_0 - h, y_0)}{h}.$$

We can notice symmetry in equation.

Solution of Poisson's Equation

The elliptic partial differential equation,

$$u_{xx} + u_{yy} = f(x, y)$$

where f(x, y) is a given function of x and y, is called the Poisson's equation.

The Poisson's equation is solved numerically by replacing the derivatives by difference expressions at the points $x = ih$, $y = jk$. Thus, we have

$$\frac{u_{i-1} - 2u_{i,j} + u_{i+1,j}}{h^2} + \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{k^2} = f(ih, jk)$$

For $h = k$, we have

$$u_{i-1} - 4u_{i,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1} = h^2 f(ih, jh)$$

The error involved in equation is $O(h^2)$.

Solution of Parabolic Equation

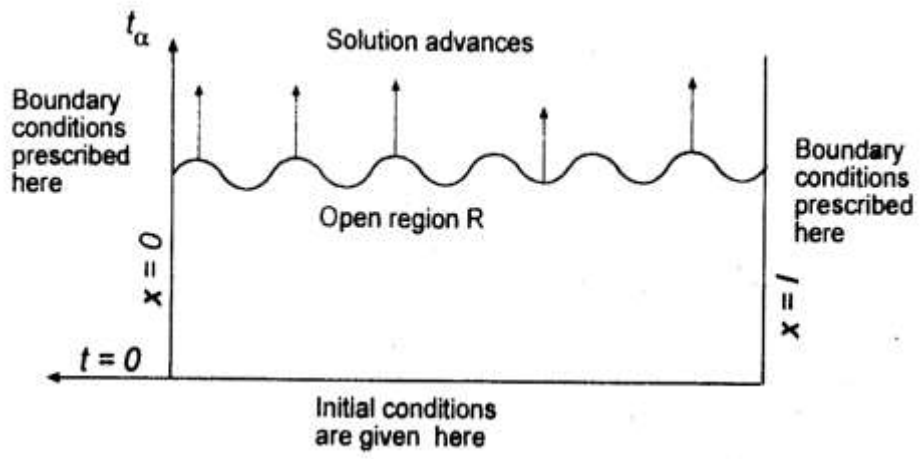
Considers the one-dimensional heat equation,

$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$; where $\alpha^2 = \frac{k}{c\rho}$

(c is the specific heat of the materials, ρ is the density and k is the thermal conductivity) is a well-known example of the parabolic equation. The solution to it is a function of x and t i.e., $u(x, t)$. It is defined for values of x from $x = 0$ to $x = I$, and for times t from $t = 0$ to $t = \infty$. The solution is not defined in a closed form (as in the case of elliptic equations) but propagates in an open-ended region from initial values satisfying the prescribed boundary condition as shown in Figure 3.1 below.

Figure 3.1: Solution domain for a PDE with prescribed initial and boundary conditions.

Result and Discussion



In this chapter we will discuss Problem Statement, we will provide it derivation, the

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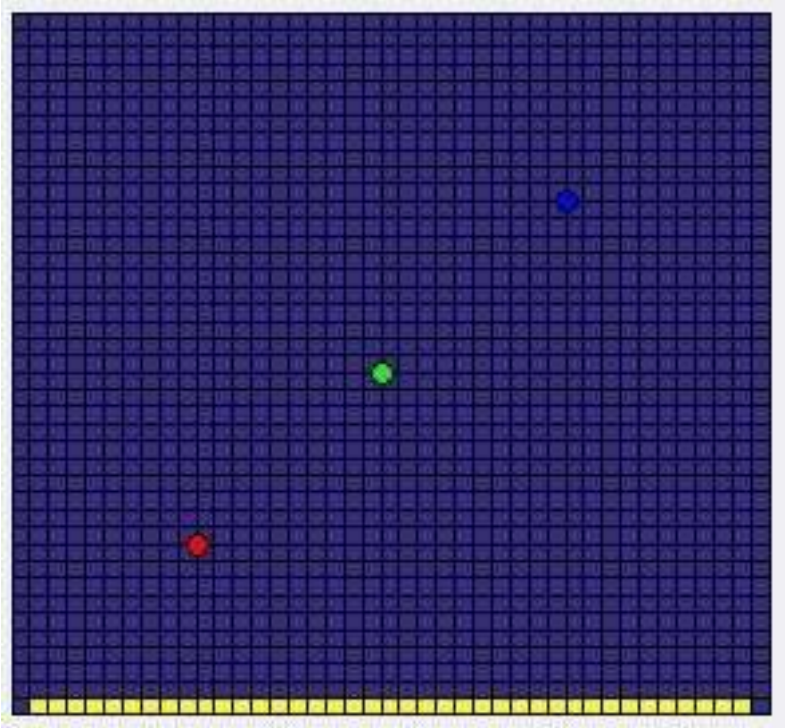
concept of finite difference method code chat flow and its result will also be investigated.

Problem Statement

Given:
Initial temperature in a 2-D plate
Boundary conditions along the boundaries of the plate. Find: Temperature in the plate as a function of time and position.

Figure 4.1: Finite grid representation of a plate with specified boundary conditions for



thermal analysis

Derivations

Solving the heat equation with central finite difference in position and forward finite difference in time using Euler method, given the heat equation in 2D,

$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$; where $\alpha^2 = \frac{k}{c\rho}$.

where,
ρ is the material density
C_p is the specific heat
K is the thermal conductivity
T (x, 0, t) = given
T (x, H, t) = given
T (0, y, t) = given
T (W, y, t) = given

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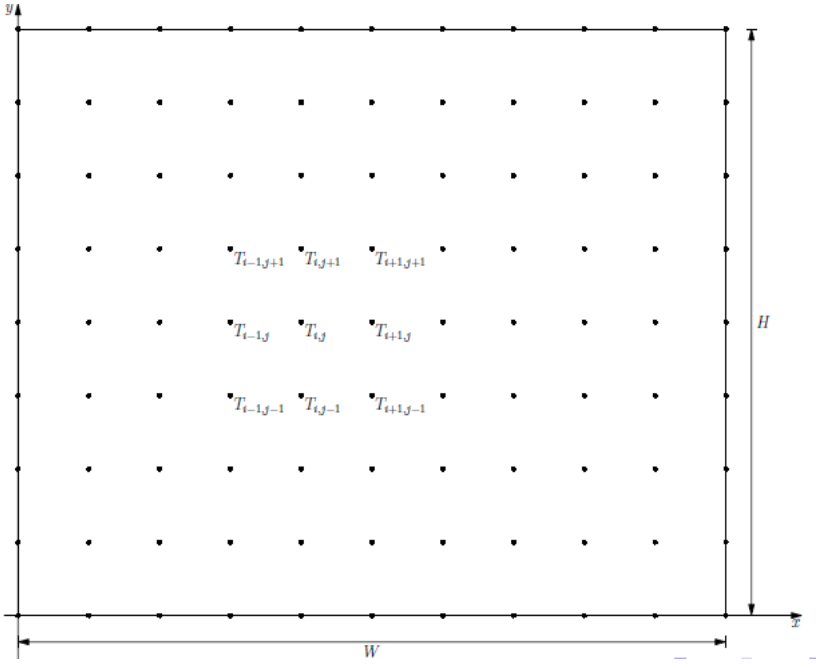
T (x, y, 0) = given

Second step is to discretize the temperatures in the plate and convert the heat equation to finite-difference form.

Figure 4.2: Computational mesh of a rectangular domain with Finite difference stencil for temperature approximation



Let's agree on that notation first

$$T^k{}_{i,j}$$

i,j=location (node number)

k=time (time step number)

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

$$\frac{T_{i,j}^{k+1} - T_{i,j}^{k}}{\Delta t} = \alpha \left[\left(\frac{T_{i-1,j}^{k} - 2T_{i,j}^{k} + T_{i+1}^{k}}{\Delta x^2}\right) + \left(\frac{T_{i,j-1}^{k} - 2T_{i,j}^{k} + T_{i,j+1}^{k}}{\Delta y^2}\right)\right]$$

$$T_{i,j}^{k+1} = T_{i,j}^{k} + \alpha \Delta t \left[\left(\frac{T_{i-1,j}^{k} - 2T_{i,j}^{k} + T_{i+1,j}^{k}}{\Delta x^2}\right) + \left(\frac{T_{i,j-1}^{k} - 2T_{i,j}^{k} + T_{i,j+1}^{k}}{\Delta y^2}\right)\right]$$

$$T_{i,j}^{k+1} = T_{i,j}^{k} - \frac{2\alpha\, \Delta t\, T_{i,j}^{k}}{\Delta x^2} - \frac{2\alpha\, \Delta t\, T_{i,j}^{k}}{\Delta y^2} + \alpha \Delta t \left[\left(\frac{T_{i-1,j}^{k} + T_{i+1,j}^{k}}{\Delta x^2}\right) + \left(\frac{T_{i,j-1}^{k} + T_{i,j+1}^{k}}{\Delta y^2}\right)\right]$$

$$T_{i,j}^{k+1} = \left(1 - \frac{2\alpha\, \Delta t}{\Delta x^2} - \frac{2\alpha\, \Delta t}{\Delta y^2}\right) T_{i,j}^{k} + \alpha \Delta t \left[\left(\frac{T_{i-1,j}^{k} + T_{i+1,j}^{k}}{\Delta x^2}\right) + \left(\frac{T_{i,j-1}^{k} + T_{i,j+1}^{k}}{\Delta y^2}\right)\right]$$

$$\left(1 - \frac{2\alpha\, \Delta t}{\Delta x^2} - \frac{2\alpha\, \Delta t}{\Delta y^2}\right) \geq 0$$

$$1 \geq \left(\frac{2\alpha}{\Delta x^2} + \frac{2\alpha}{\Delta y^2}\right) \Delta t.$$

Where, $\Delta t \leq \frac{1}{2\alpha\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right)}$ is the stability condition and the final equation is

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$$T_{i,j}^{k+1} = \left(1 - \left(\frac{2\alpha}{\Delta x^2} + \frac{2\alpha}{\Delta y^2}\right)\Delta t\right) T_{i,j}^{k} + \alpha \Delta t \left[\left(\frac{T_{i-1,j}^{k} + T_{i+1,j}^{k}}{\Delta x^2}\right) + \left(\frac{T_{i,j-1}^{k} + T_{i,j+1}^{k}}{\Delta y^2}\right)\right].$$

Code Flow Chat

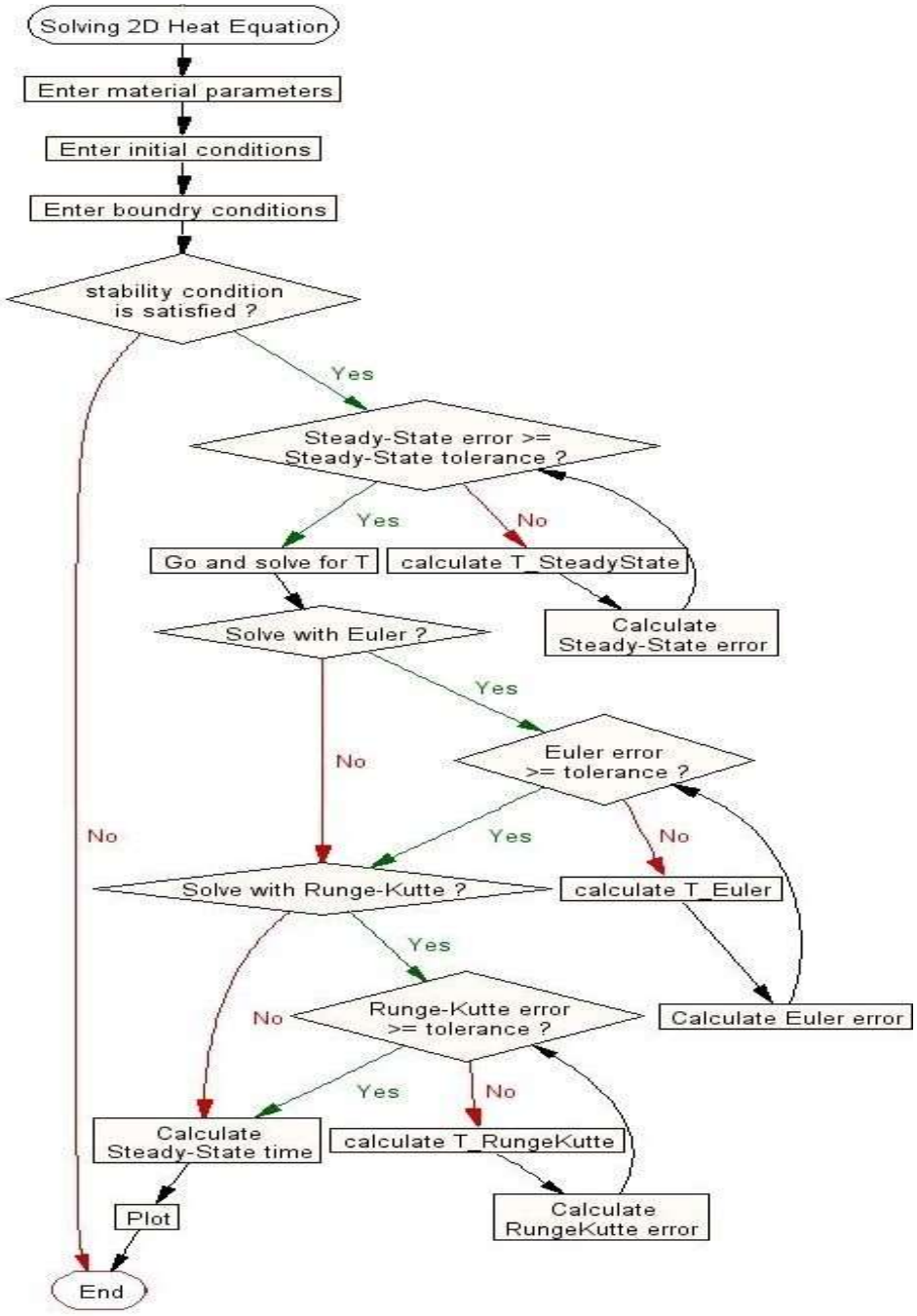

Figure 4.3: Flow diagram of Finite difference heat equation solver with convergence and error analysis

Results:

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Different Materials with same conditions

Aluminum

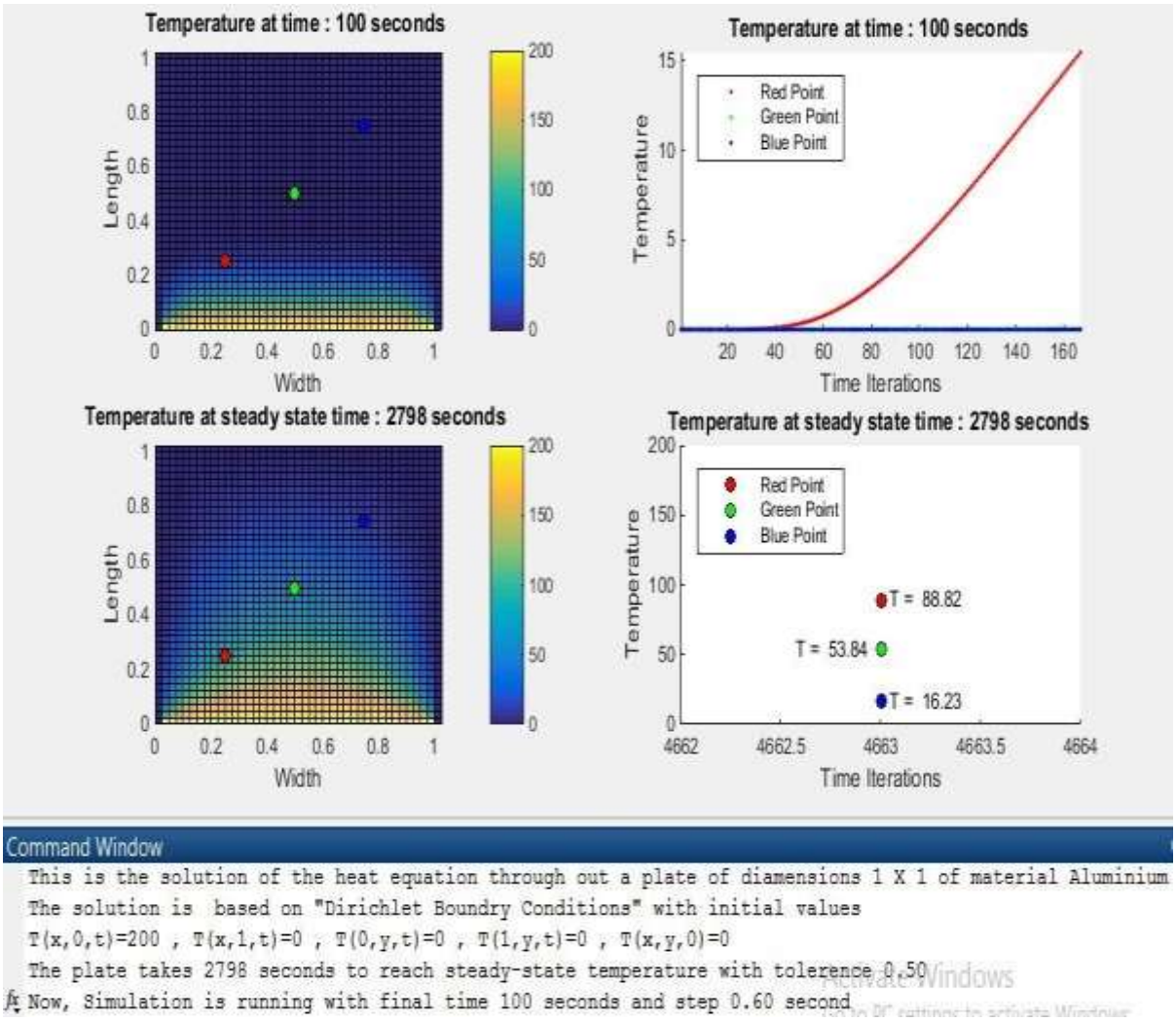

Figure 4.4: Temperature field in a 1 m × 1 m aluminum plate under Dirichlet boundary conditions at t = 100 s and steady state (t = 2798 s)

The temperature evolution within the 1×1 aluminum plate under Dirichlet boundary conditions (with the top edge fixed at 200 °C, the bottom edge at 0 °C, and the left/right edges at 0 °C) is presented as shown in figure 4.4. The upper-left plot shows the temperature distribution at t = 100 seconds, where heat has only begun to diffuse from the top boundary into the interior, resulting in a shallow thermal gradient and a maximum temperature well below 200 °C. In contrast, the lower-left plot illustrates the steady-state condition reached at 2798 seconds, where the temperature field has fully stabilized, exhibiting a smooth, monotonic decrease from the hot top edge to the cold bottom edge. Compared to the subsequent trials with copper and silver, aluminum's slower thermal diffusivity causes a markedly longer time to reach steady state, confirming that heat diffusion in this material is significantly more gradual.

Copper

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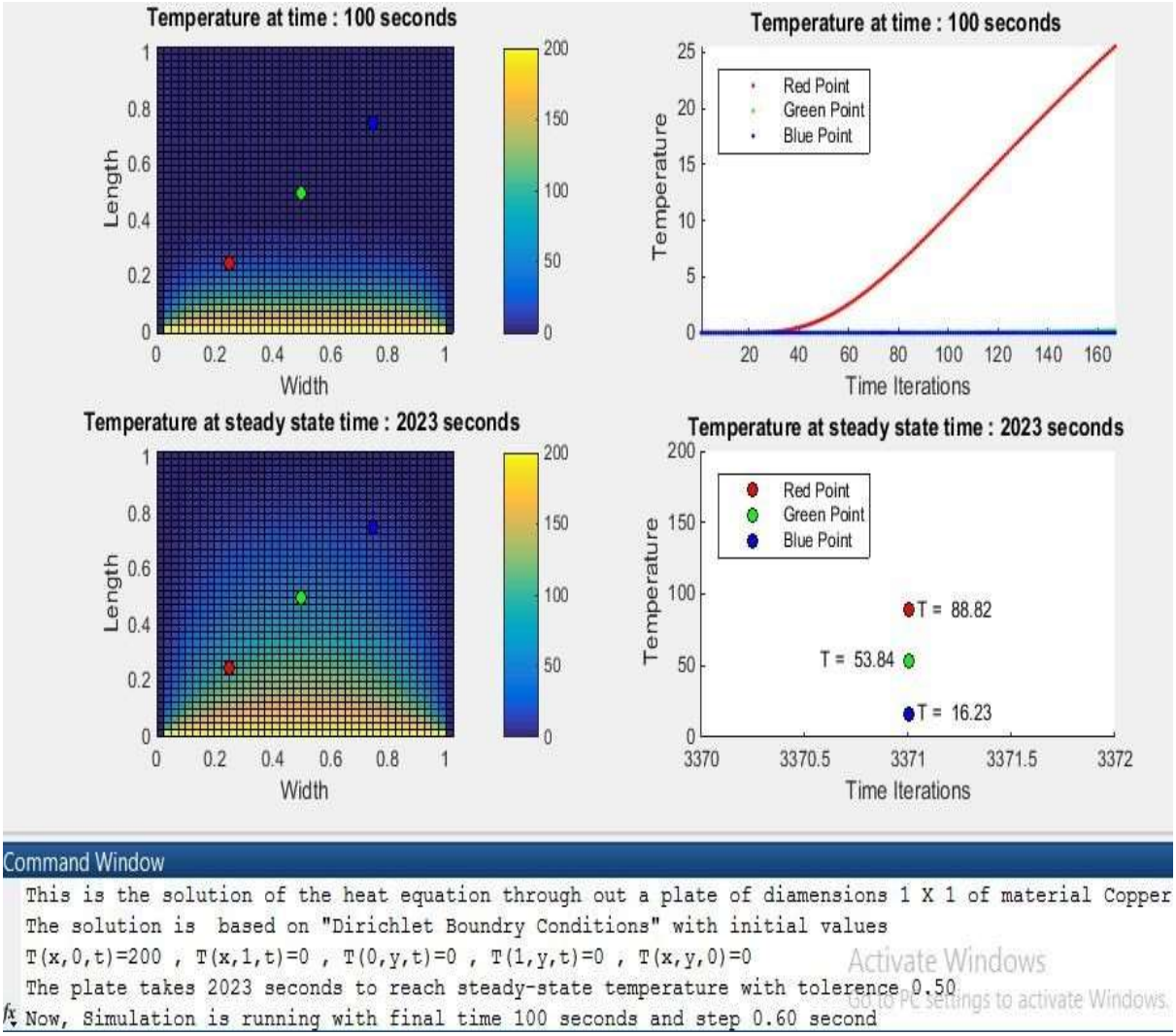
Figure 4.5: Temperature distribution in a 1×1 copper plate at t = 100 s and steady state (t = 2023 s) under Dirichlet boundary conditions



The temperature evolution within a 1×1 meter copper plate subjected to Dirichlet boundary conditions, where the top edge (y = 0) is fixed at 200 °C, the bottom edge (y = 1) at 0 °C, and both left (x = 0) and right (x = 1) edges at 0 °C as shown in figure 4.5. The upper plot shows the temperature distribution at t = 100 seconds, where heat has diffused from the hot top boundary downward into the interior, creating a visible thermal gradient that remains far from equilibrium. The color bar indicates temperatures ranging from cool (blue) near the bottom to warm (green/yellow) near the top, with red representing the highest temperatures. The lower plot illustrates the steady-state condition reached at 2023 seconds, where the temperature field has fully stabilized into a smooth, monotonic transition from 200 °C at the top edge to 0 °C at the bottom edge, with isotherms running horizontally across the width. Compared to aluminum (which required 2798 seconds), copper reaches steady state more quickly due to its higher thermal diffusivity, as evidenced by the broader heat penetration at

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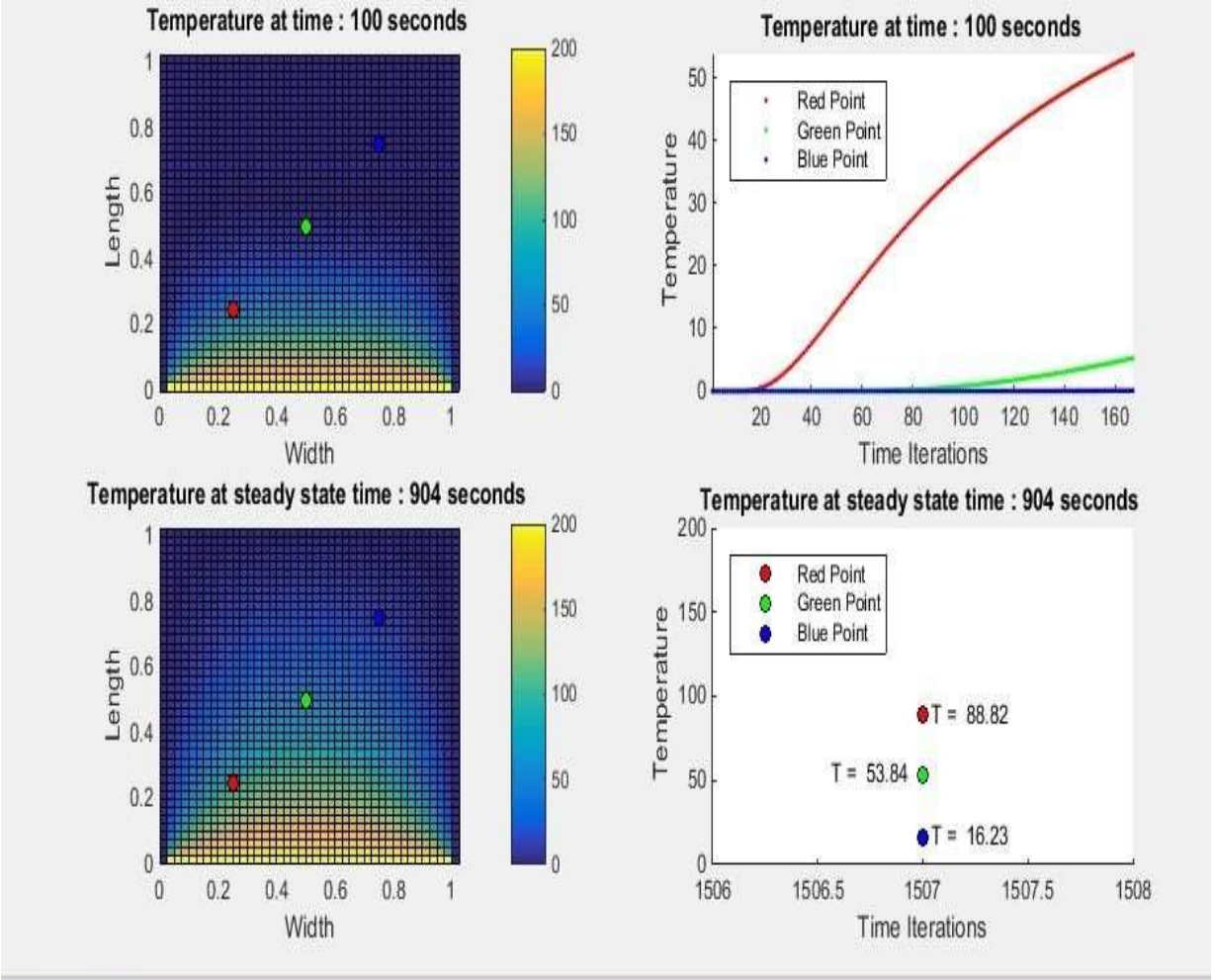
100 seconds and the shorter overall time to equilibrium.

Silver

Figure 4.6: Temperature distribution in a 1×1 silver plate at t = 100 s and steady state



(t = 904 s) under dirichlet boundary conditions

The figure 4.6 shows temperature evolution in a 1×1 silver plate under Dirichlet boundary conditions (top edge at 200 °C, other edges at 0 °C). At t = 100 seconds, heat has diffused substantially downward from the top, reflecting silver's high thermal diffusivity. The steady-state condition is reached at 904 seconds, displaying a smooth vertical gradient from 200 °C to 0 °C. Compared to aluminum (2798 seconds) and copper (2023 seconds), silver reaches equilibrium fastest due to its superior thermal conductivity. The contour and line plots together confirm rapid heat penetration and convergence to steady temperature values.

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Different boundary conditions with the same initial (200 deg C)

Trial A

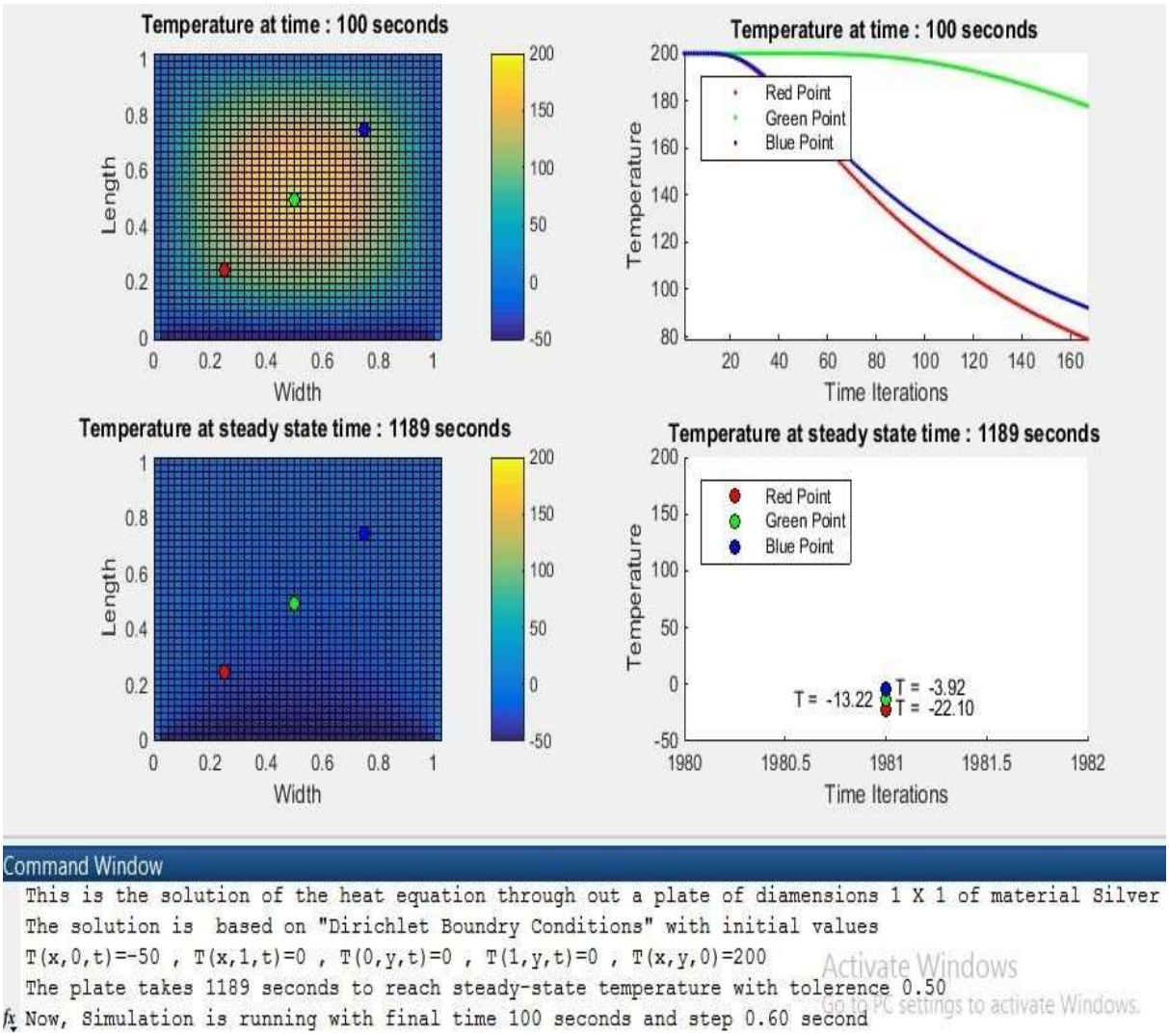

Figure 4.7: Temperature distribution in a silver plate with inverted initial and boundary conditions

The figure 4.7 presents temperature evolution in a 1×1 silver plate under Dirichlet boundary conditions, where the top edge is fixed at -50 °C, bottom and side edges at 0 °C, and the initial uniform temperature is 200 °C. At t = 100 seconds, the plate shows significant cooling from the top downward, with heat diffusing upward toward the cold boundary. The steady-state condition is reached at 1189 seconds, where the temperature field stabilizes with recorded values of -13.22 °C and -22.10 °C at specific locations. Unlike the previous case with a hot top edge (200 °C), this scenario involves cooling from an initially hot plate to a cold top boundary. Silver's high thermal diffusivity enables relatively fast equilibrium despite the inverted thermal gradient.

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Trial B

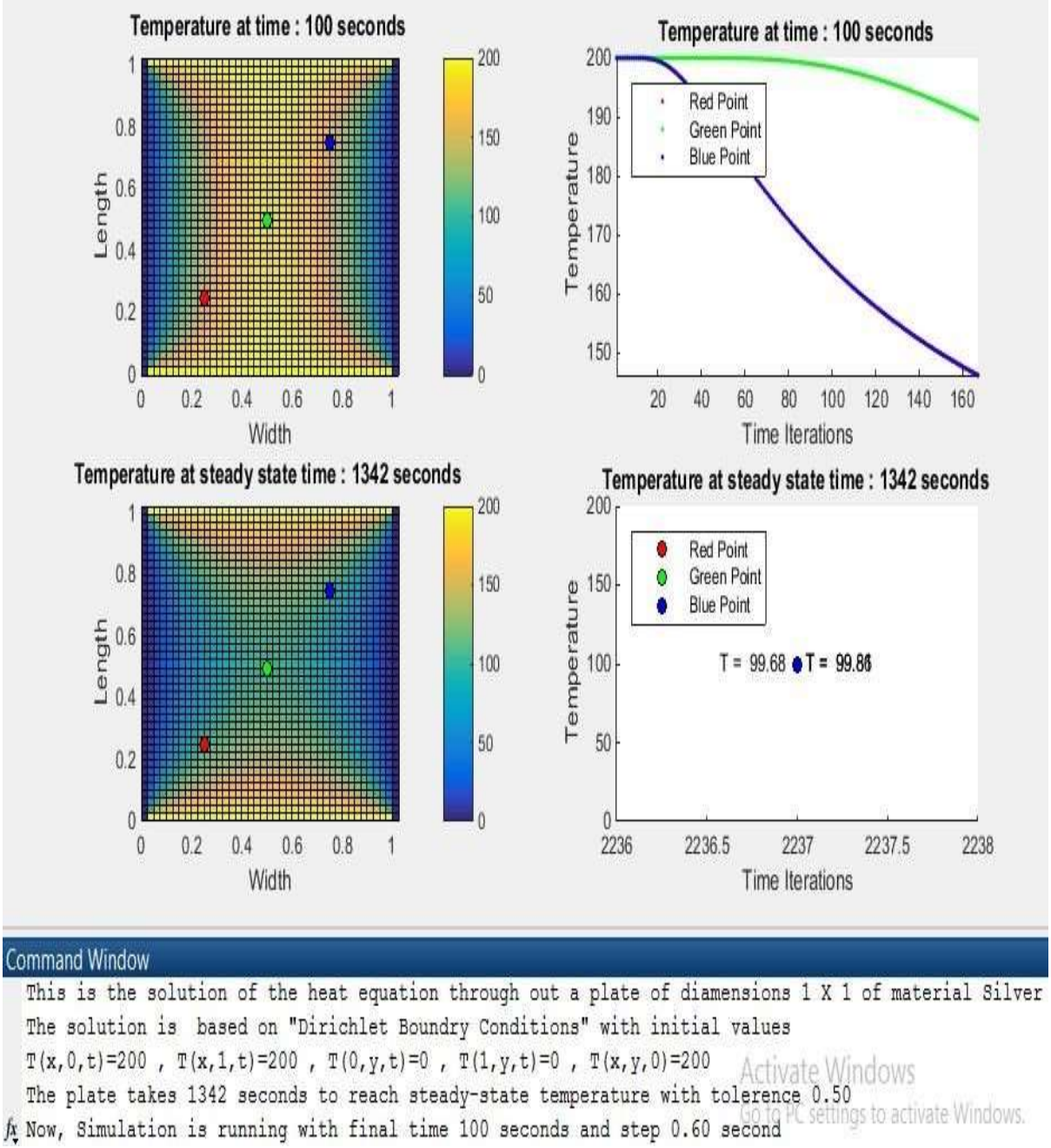

Figure 4.8: Temperature distribution in a silver plate with both top and bottom edges heated

The figure 4.8 presents temperature evolution in a 1×1 silver plate with Dirichlet boundary conditions, where both top and bottom edges are fixed at 200 °C, side edges at 0 °C, and initial temperature at 200 °C. At t = 100 seconds, heat diffuses inward symmetrically from top and bottom, creating a cooler central region near the side walls, with steady state reached at 1342 seconds. Silver's high thermal diffusivity

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enables symmetric thermal stabilization, resulting in a temperature field that gradually decreases from the heated horizontal edges toward the cooler vertical boundaries.

Trial C

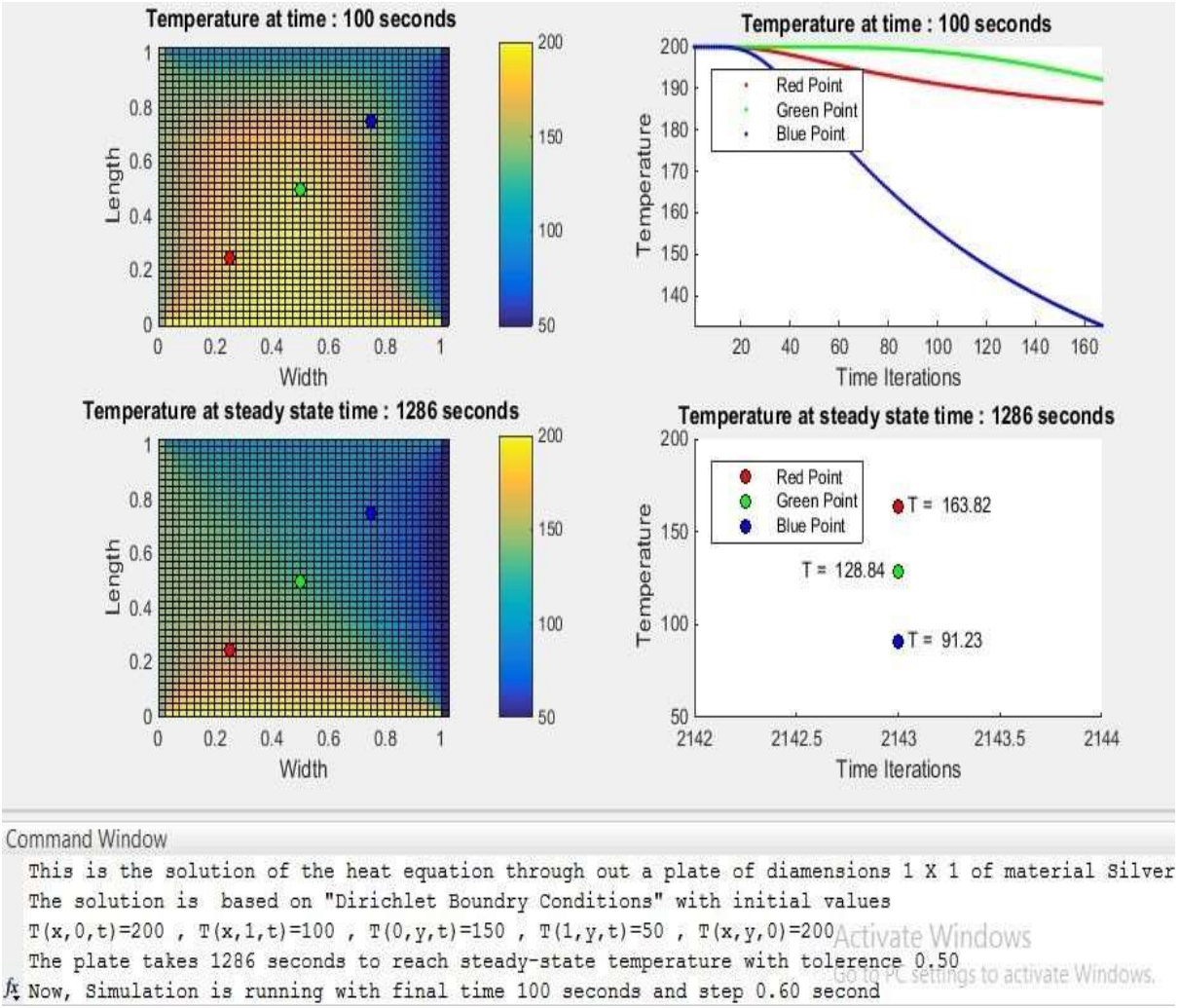

Figure 4.9: Temperature distribution in a silver plate with asymmetric Dirichlet boundary conditions

The figure 4.9 illustrates temperature evolution in a 1×1 silver plate under asymmetric Dirichlet boundary conditions, with the top edge at 200 °C, bottom at 100 °C, left edge at 150 °C, and right edge at 50 °C, starting from an initial uniform temperature of 200 °C. At t = 100 seconds, heat diffuses unevenly across the plate, creating a complex thermal gradient that reflects the four different boundary temperatures, with steady state reached at 1286 seconds. Silver's high thermal diffusivity facilitates relatively rapid convergence to equilibrium, resulting in a smooth temperature field that transitions from the hottest corner (top-left) to the coolest corner (bottom-right).

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Different initial conditions with the same boundaries (200, 0, 0 & 0)

Trial A

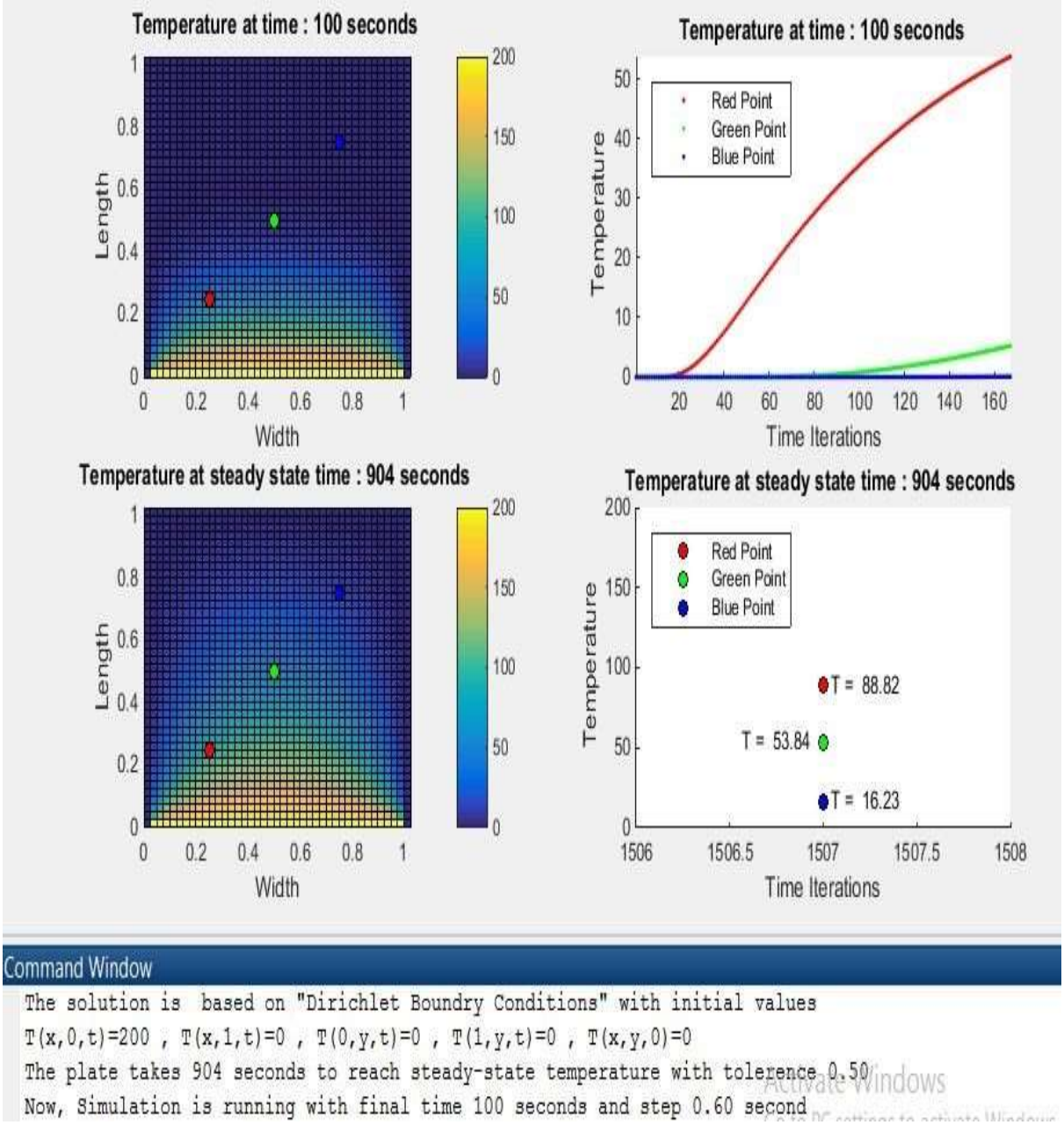

Figure 5.1: Temperature distribution in a plate at t = 100 s and steady state (904 s) under Dirichlet conditions

The temperature evolution in a 1×1 plate with Dirichlet boundary conditions, where the top edge is fixed at 200 °C, all other edges at 0 °C, and initial temperature at 0 °C. At t = 100 seconds, heat has diffused downward from the top boundary, creating a visible thermal gradient, while the steady-state condition at 904 seconds shows a fully developed linear temperature decrease from top to bottom as shown in Figure 5.1. The

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accompanying line plots track temperature at specific points over time iterations, confirming monotonic heating and convergence to steady values.

Trial B

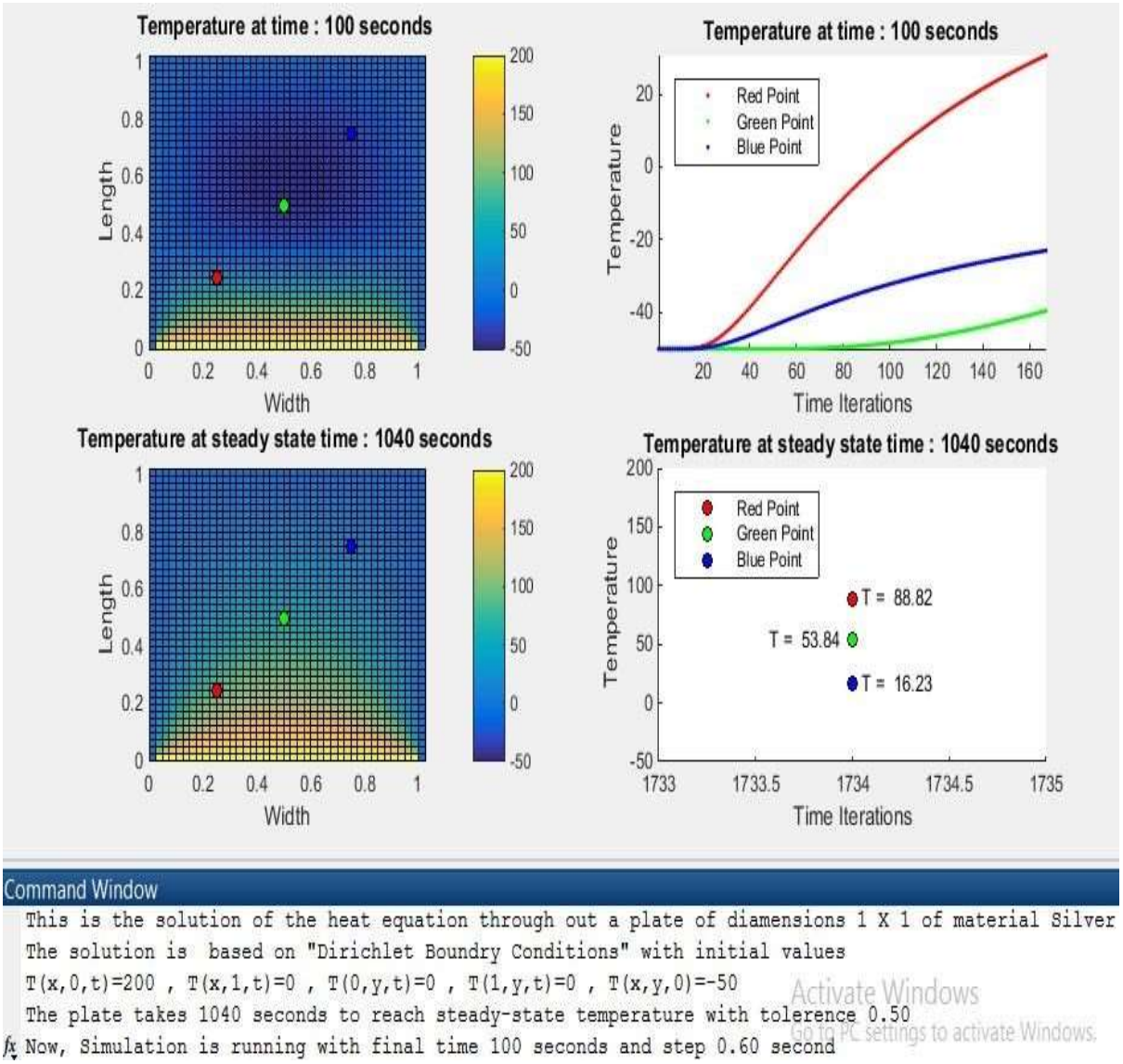

Figure 5.2: Temperature evolution in a silver plate from a cold initial state to steady state

The Figure 5.2 presents temperature evolution in a 1×1 silver plate with the top edge at 200 °C, other edges at 0 °C, and an initial temperature of -50 °C; at t = 100 seconds, heat has penetrated the upper region while the bottom remains cold. Steady state is reached at 1040 seconds, showing a smooth vertical gradient from 200 °C to 0 °C, with silver's high diffusivity enabling rapid warming despite the cold initial condition

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Trial C

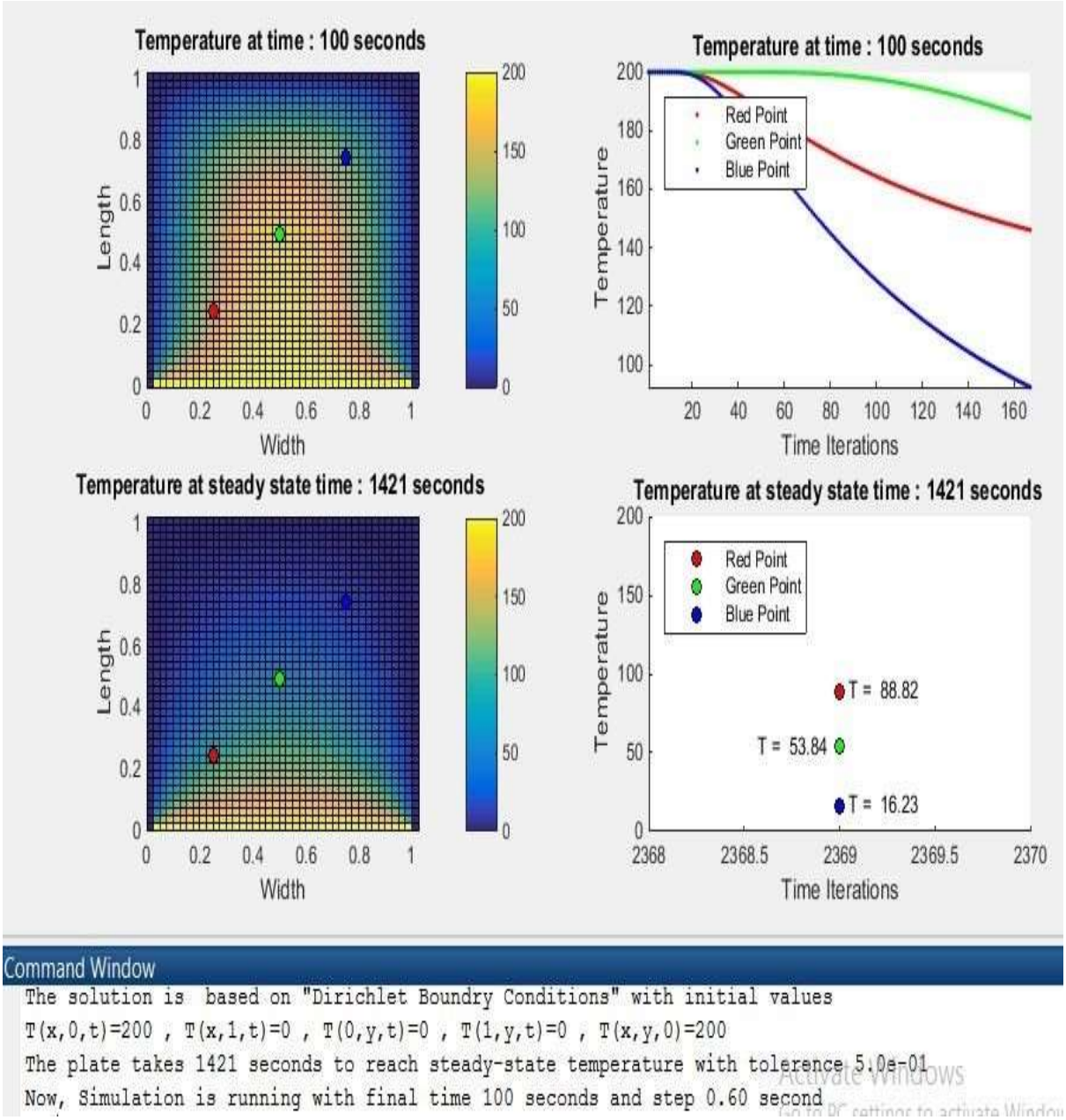

Figure 5.3: Temperature distribution in a cooling plate from uniform 200°C to steady state

Figure 5.3 shows a 1×1 plate with Dirichlet boundary conditions (top at 200°C, other edges at 0°C) starting from a uniform initial temperature of 200°C; at t = 100 seconds, the interior has begun cooling from the bottom and sides while the top remains hot. Steady state is reached at 1421 seconds, displaying a smooth vertical gradient from 200°C at the top to 0°C at the bottom, with the line plots confirming progressive cooling at monitored points.

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Runge-Kutta and Euler in the same conditions

Trial A

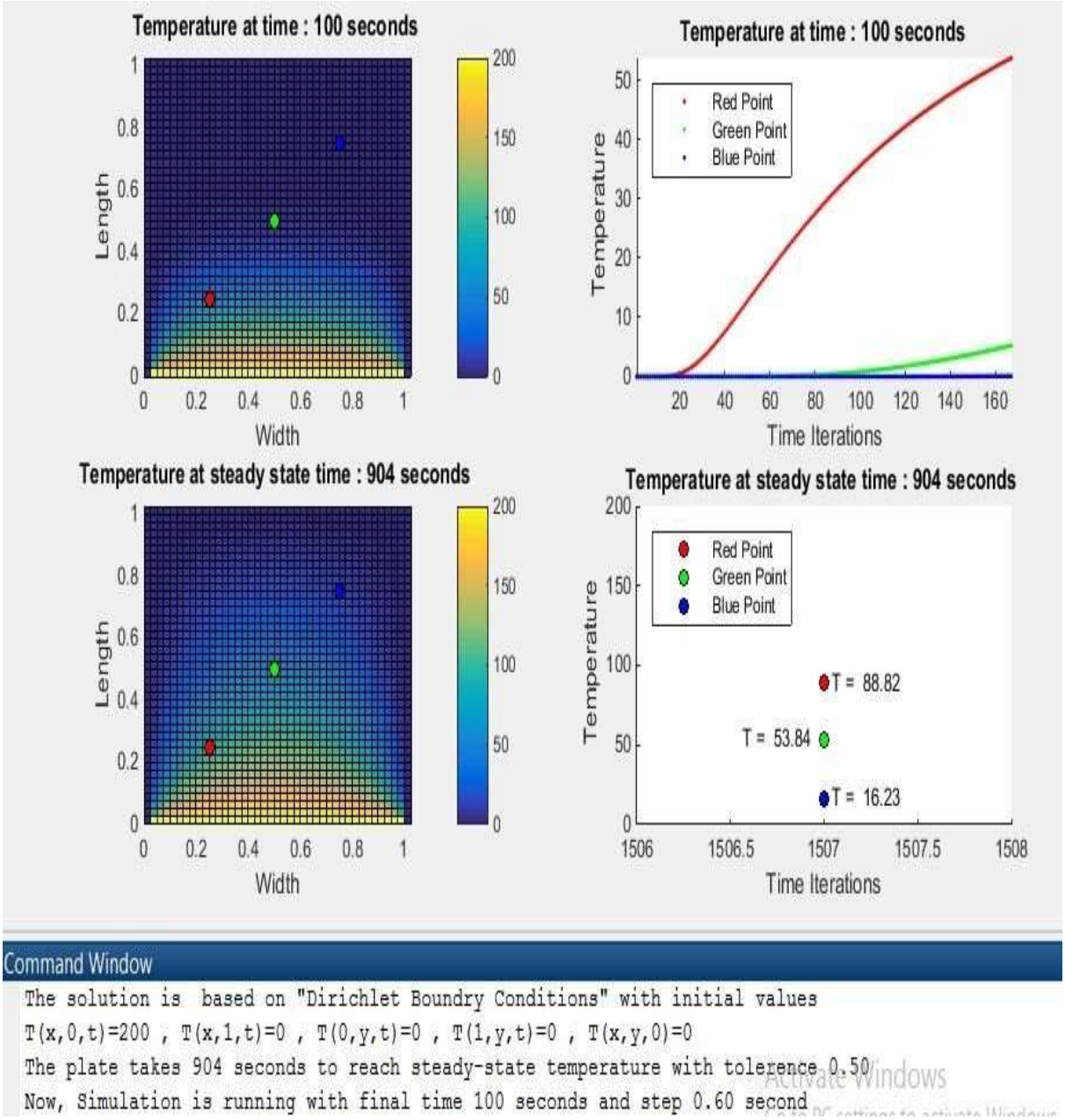

Figure 5.4: Temperature evolution in a plate from cold start to steady state under Dirichlet conditions

The figure 5.4 presents temperature evolution in a 1×1 plate with the top edge fixed at 200 °C, all other edges at 0 °C, and an initial temperature of 0 °C; at t = 100 seconds, heat has diffused downward from the top boundary, creating a pronounced thermal gradient. Steady state is achieved at 904 seconds, showing a fully developed linear temperature decrease from 200 °C at the top to 0 °C at the bottom, with the accompanying line plots confirming monotonic heating and convergence at monitored points.

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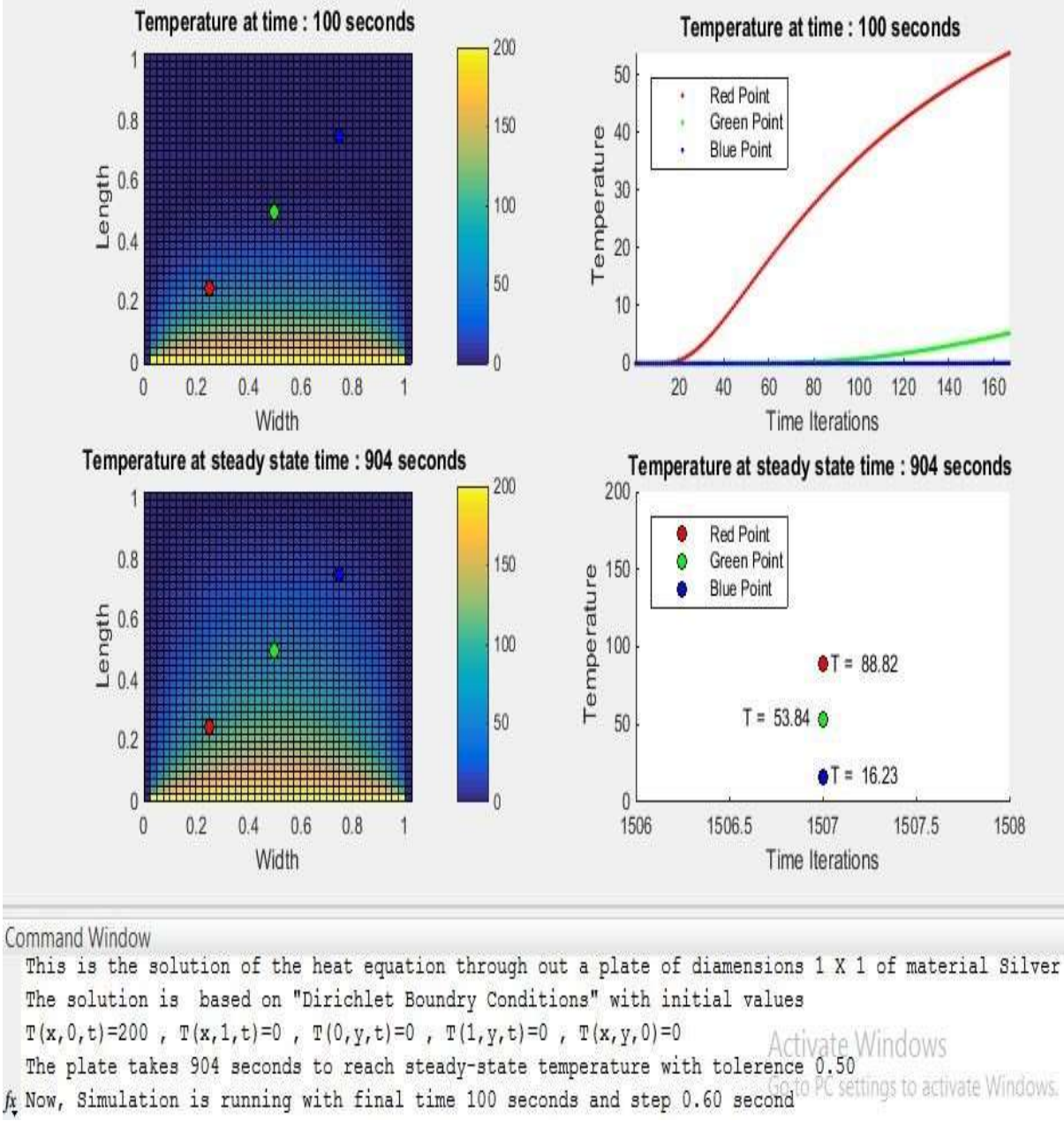

Trial B

Figure 5.5 Transient and steady-state temperature distribution in a silver plate (Dirichlet conditions)

The figure 5.5 illustrates heat diffusion in a 1×1 silver plate with the top edge at 200 °C, other edges at 0 °C, and initial temperature of 0 °C; at t = 100 seconds, heat has penetrated the upper portion while the lower half remains cool. Steady state is achieved at 904 seconds, showing a linear vertical gradient from 200 °C to 0 °C, with silver's high thermal diffusivity enabling rapid equilibrium.

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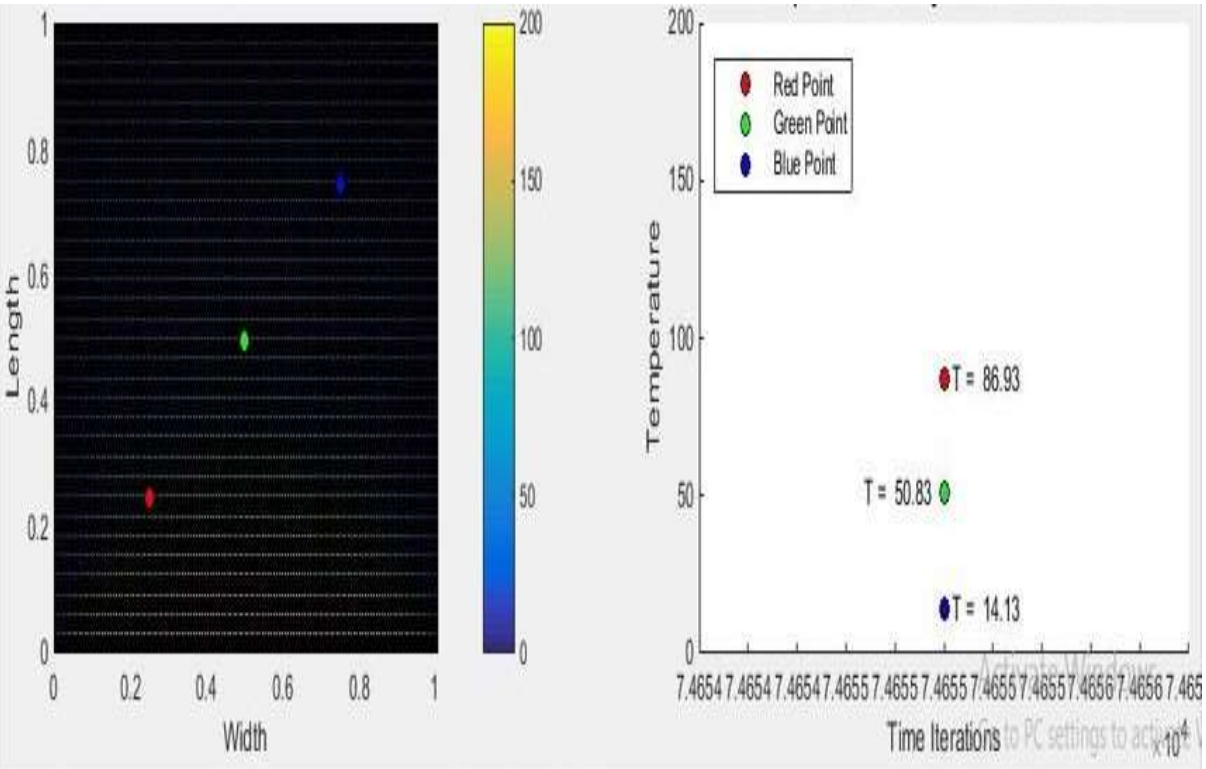
Different number of nodes in x and y directions

Trial A




```
Lx= 1;                          % plate width (m)
Ly= 1;                          % plate length (m)
Nx=200;                          % nodes in x direction
Ny=200;                          % nodes in y direction

tolerence_ss=0.000001;          % tolerence for steady state section
```

Figure 5.6: Numerical simulation results of steady-state temperatures in a 1×1 plate

Figure 5.6 presents numerical simulation results for a 1×1 plate with initial condition 0°C and Dirichlet boundary conditions (top at 200°C, other edges at 0°C), solved using the Euler method to steady state only due to limited computational resources. Using 200×200 nodes (15-minute runtime), the center temperature is 50.83°C (error 1.66%), a significant improvement over the 40-node trial which gave 53.84°C (error 7.68%), confirming that mesh refinement enhances accuracy.

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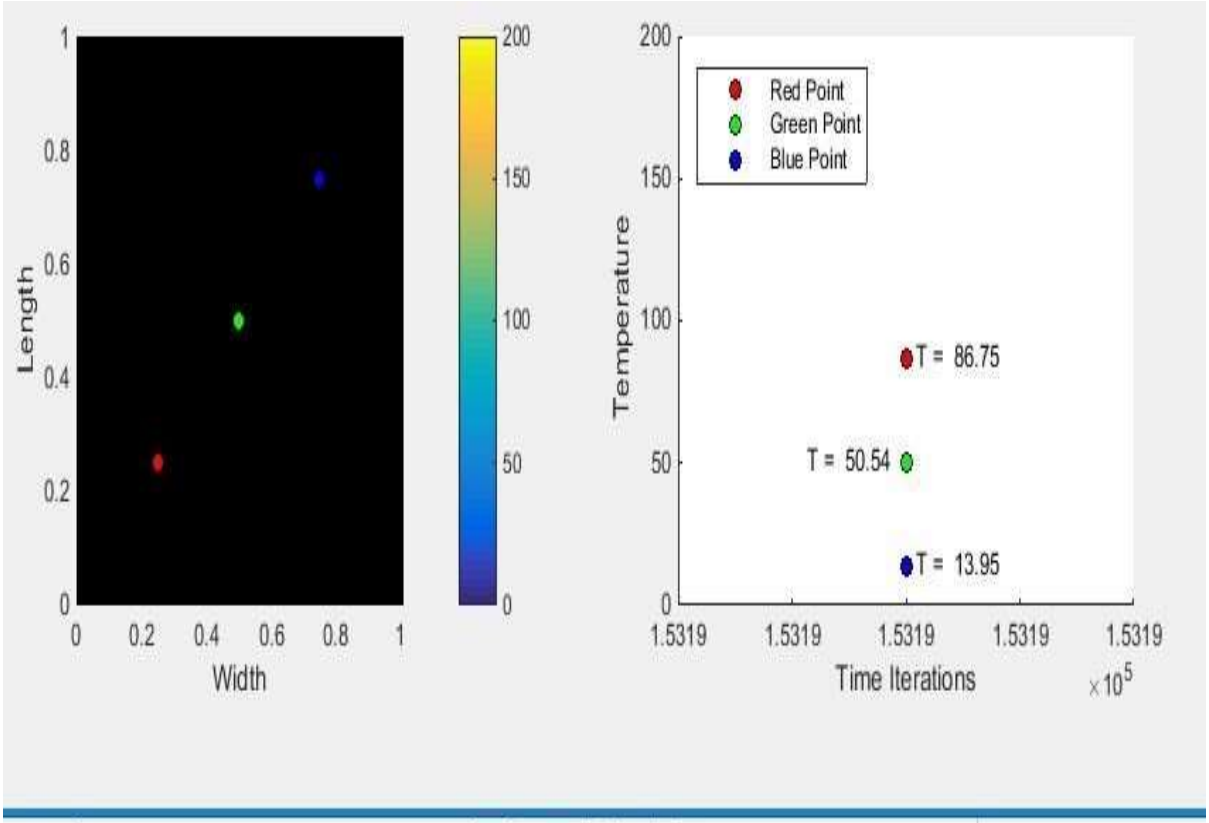
Trial B




```
Lx= 1;                          % plate width (m)
Ly= 1;                          % plate length (m)
Nx=300;                          % nodes in x direction
Ny=300;                          % nodes in y direction

tolerence_ss=0.000001;           % tolerence for steady state section
```

Figure 5.7: Steady-state temperature distribution in a 1×1 plate with 300×300 mesh refinement

The Figure 5.7 presents steady-state simulation results for a 1×1 plate with initial condition 0°C and Dirichlet boundary conditions (top at 200°C, other edges at 0°C), solved using the Euler method on a refined 300×300 mesh (tolerance 1×10^{-6}). The computed temperatures at three monitoring points are 86.75°C, 50.54°C (center), and 13.95°C, where the center value shows improved accuracy (error 1.08%) compared to 200×200 mesh (50.83°C, error 1.66%) and 40-node mesh (53.84°C, error 7.68%), confirming that further mesh refinement enhances solution precision.

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More results with different conditions

Trial A

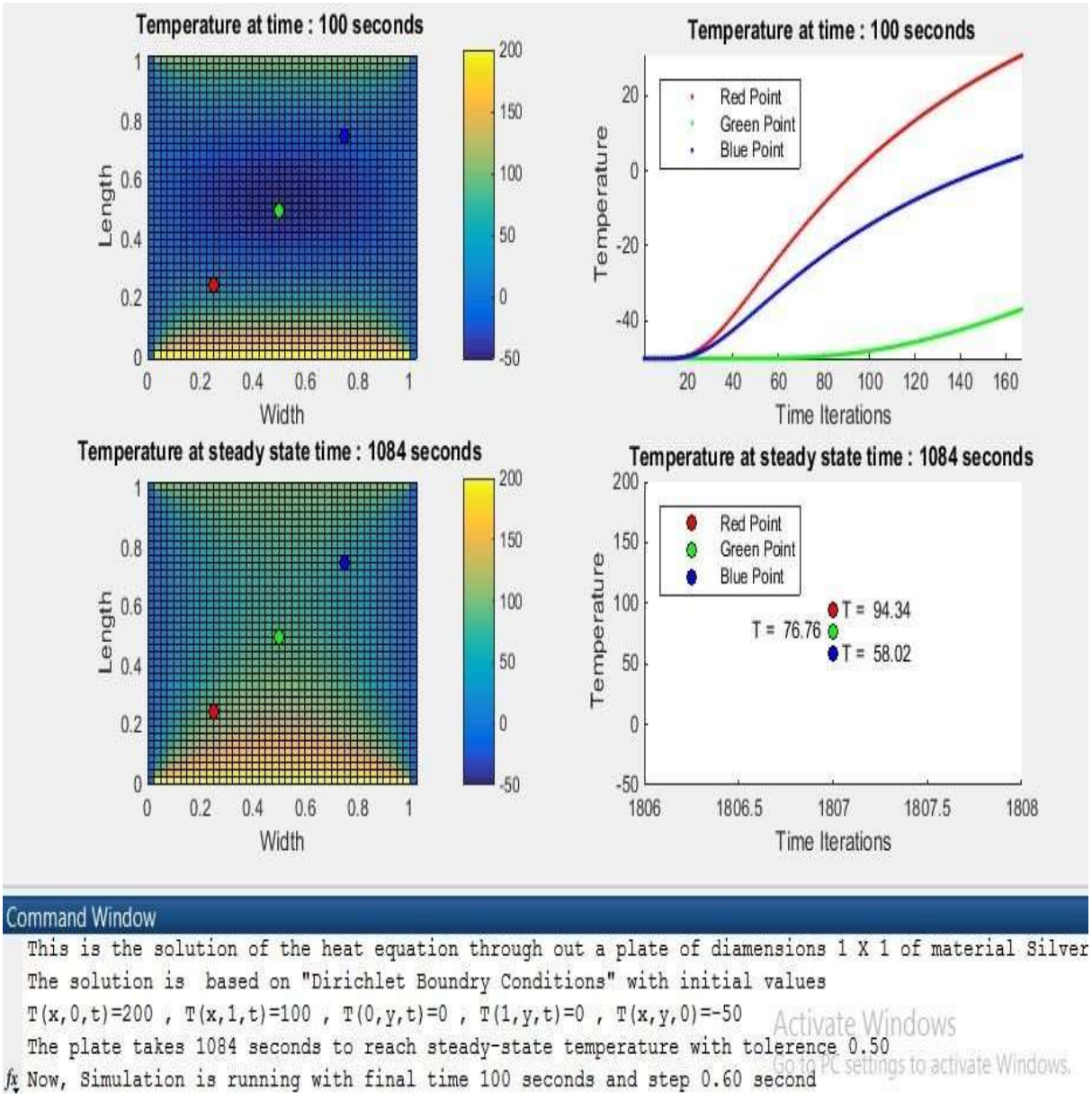

Figure 5.8: Temperature evolution in a silver plate with asymmetric boundary conditions and cold initial state

Figure 5.8 illustrates heat diffusion in a 1×1 silver plate with Dirichlet boundary conditions (top at 200°C, bottom at 100°C, sides at 0°C) starting from an initial uniform temperature of -50°C; at t = 100 seconds, heat has penetrated from the top and bottom boundaries while side regions remain cold. Steady state is achieved at 1084 seconds, showing a smooth thermal gradient from 200°C at the top to 100°C at the bottom, with the line plots confirming temperature evolution at three monitored points

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Trial B

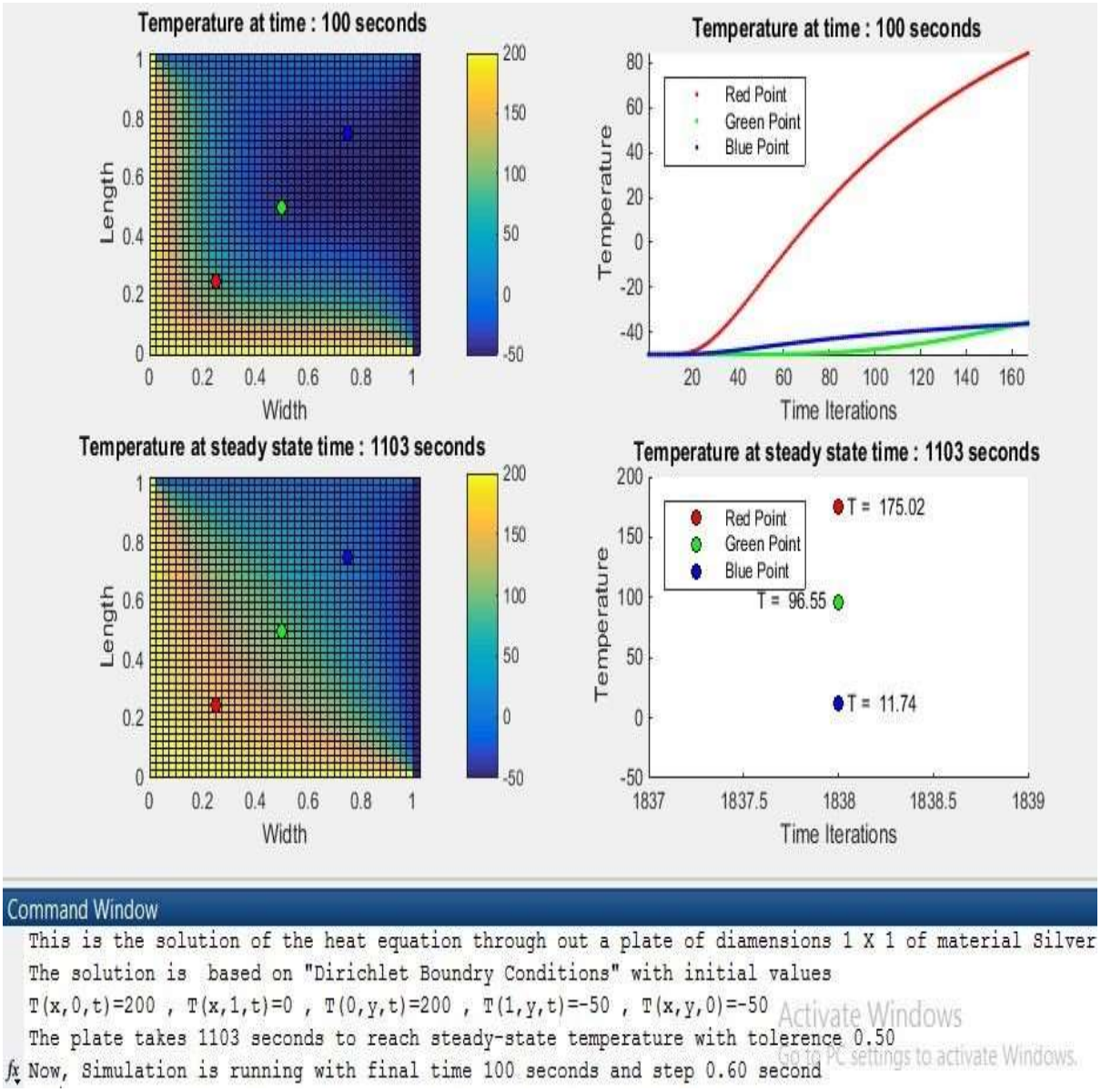

Figure 5.9: Temperature evolution in a silver plate with mixed hot and cold boundary conditions

Figure 5.9 presents heat diffusion in a 1×1 silver plate with Dirichlet boundary conditions (top at 200°C, bottom at 0°C, left edge at 200°C, right edge at -50°C) starting from an initial uniform temperature of -50°C; at t = 100 seconds, heat from the top and left boundaries has penetrated the plate while the right side remains very cold. Steady state is reached at 1103 seconds, showing a complex thermal gradient that transitions from the hot top-left corner to the cold bottom-right corner, with line plots confirming temperature evolution at monitored points.

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Trial C

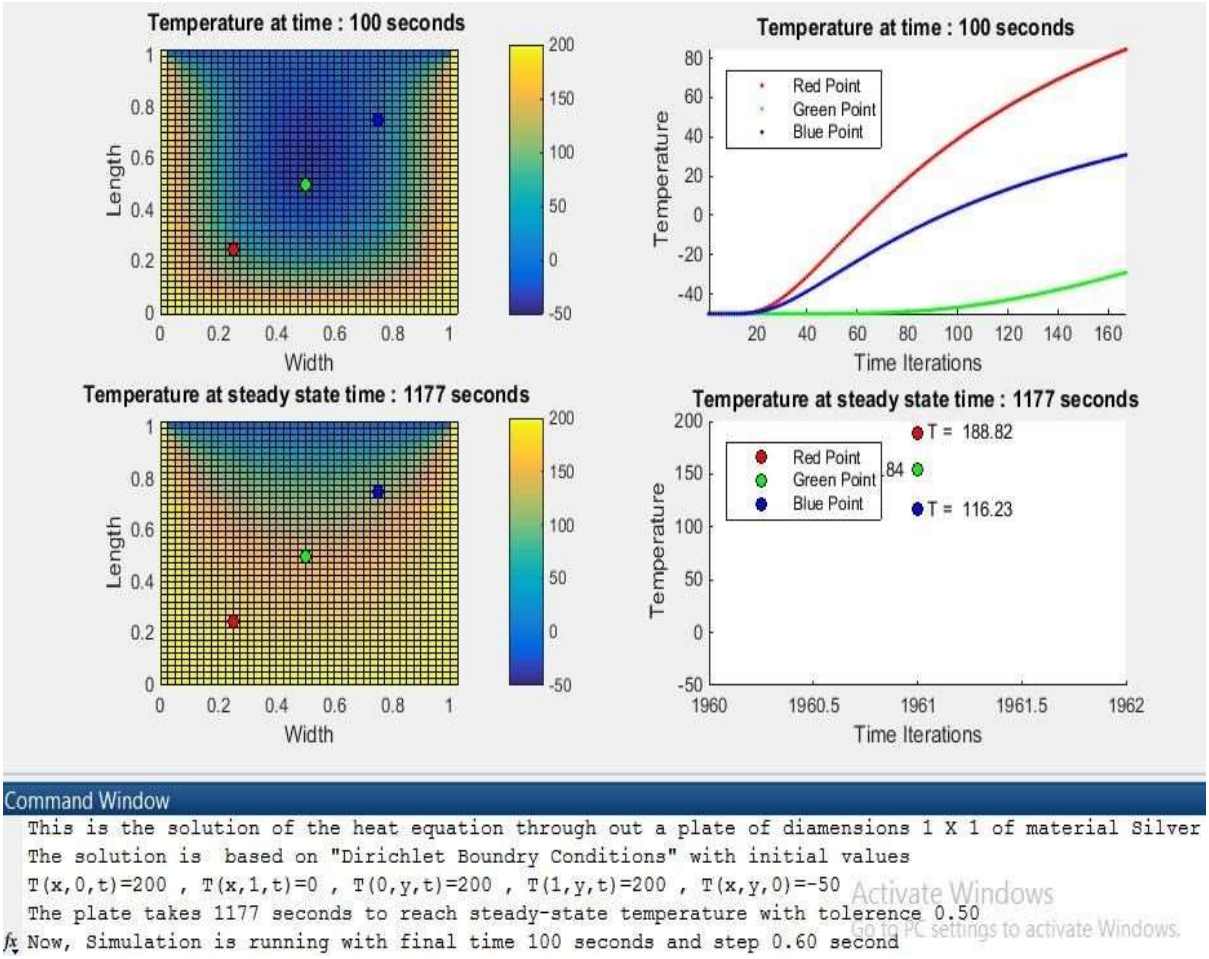

Figure 5.10: Temperature evolution in a silver plate with three hot edges and a cold initial state

Figure 5.10 shows heat diffusion in a 1×1 silver plate with Dirichlet boundary conditions (top at 200°C, bottom at 0°C, left and right edges at 200°C) starting from an initial uniform temperature of -50°C; at t = 100 seconds, heat from the top and side boundaries has penetrated the plate while the bottom remains cold. Steady state is reached at 1177 seconds, with recorded temperatures of 188.82°C and 116.23°C at monitored points, reflecting a complex gradient from the hot top-left corner toward the cold bottom edge.

Conclusion

This approach successfully utilizes the finite difference technique to solve the 2D heat equation and investigate heat diffusion within a square plate. The implemented MATLAB code allows users to input thermal conductivity, specific heat, and density, enabling simulation of various materials including aluminum (2798 s to steady state), copper (2023 s), and silver (904–1342 s depending on boundary conditions). The code supports user-defined Dirichlet boundary conditions and calculates steady-state time

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with adjustable error tolerance. Mesh refinement studies demonstrated improved accuracy: increasing nodes from 40×40 (center temperature 53.84°C, error 7.68%) to 200×200 (50.83°C, error 1.66%) to 300×300 (50.54°C, error 1.08%) confirmed convergence toward the analytical optimum of 50°C, albeit with increased computational time (15 minutes for 200×200 nodes). The algorithm offers both Euler and 2nd-order Runge-Kutta methods for time discretization alongside central finite differences for spatial derivatives. Simulation results across multiple boundary configurations—including single-edge heating, dual-edge heating, asymmetric conditions, and cold initial states—effectively illustrated the dynamics of heat diffusion. Overall, this project provides a flexible and customizable tool for analyzing 2D heat distribution across various materials and boundary conditions, with mesh refinement validation confirming solution accuracy.

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