Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about
Volume 4, Issue 2 (2026)
https://doi.org/10.5281/zenodo.18865385

Online ISSN 3007-3197
Print ISSN 3007-3189

http://amresearchreview.com/index.php/Journal/about

An Exploration Of Mixed Convection Casson Nano Fluid Motion Over An Extending Sheet With Cattaneo-Christov Heat Flux

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ABSTRACT

In order to cope with the increasing requirement for efficient thermal transportation in compact heat exchanges, this work has been under taken to investigate theoretically the various characteristics for Casson nanofluids flow problem. The base liquid is constituted to have more thermal conductivity with mild homogenous mixing of nano species. The steady two-dimensional flows is considered over an extending sheet under the effect of magnetic field applied normal to the surface. The buoyancy effects have also been examined. The heat transfer involves convection and Cattaneo-Christove heat flux. The coupled non-linear partial differential equations are considered in the formulation of the problems that have been transformed by employing appropriate similarity functions. Numerical solutions have been sought for the resulting ordinary differential equation by using the Runge-Kutta Method with shooting technique. A robust coding is developed in Matlab script. The validation of the code of numerical scheme is established with a close agreement among the current and previously existing results as special case. Results for physical quantities of interest have been computed for vide ranges of the pertinent parameters. The flow speed is inhibited against increments of the parameters of magnetic field and porosity. The temperature rises up directly with parameters of Brownian motion and thermophoresis. Representative results have been demonstrated in tabular and graphical form discussed and these are discussed in detail.

Keywords: Casson Nano-Fluids, Cattaneo-Christove Heat Flux, Mixed Convection, Brownian Motion.

Introduction

Fluid dynamics deals with the motion of fluids. Its application are related to different engineering and technological processes. The major kinds of fluids are Newtonian

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fluid and non-Newtonian fluids. Casson fluids are non-Newtonian fluids. Casson fluid is a shear thinning liquid with a zero viscosity at zero rate of shear, an infinite viscosity at zero rate of shear, and a yield stress below which no flow occurs. Magnetohydrodynamics is the research of the magneto strictive behavior of materials of conductive liquids. The term "magnetohydrodynamics" is derived from the prefix magneto-, which means magnetic field, the prefix hydro-, which means water, and the prefix dynamics, which means movement. Heat transfer and fluid flow are strongly intertwined. The three modes of heat transfer are convection, conduction, and radiation. Fourier's law governs heat transfer. However, cattaneo-charistove diffusion is a non-Fourier law that is thought to be more effective.

The Cattaneo-Christove thermal conduction method is used to predict heat transfer in viscoelastic flow caused by an exponentially stretching surface. The following are the major spots of this research: The flow boundary surface in viscous incompressible is thinner than in viscous fluid. Most of the industrial and mechanical processes require efficient heat transportation. The emerging marcher and small size technology are in the dire need of effective heat transfer techniques. In traditional there are many methods for heat transfer such as cooling of air and water fins and fining etc. However, there are difficulties and lack of efficiency in using such convectional techniques. Nano fluids as introduced by Choi in 1995. These fluids provide a promising solution for effective heat transfer by increasing the thermal conductivity base fluid with the diffusion of nanoparticles. Recently researcher investigating various aspects of nano fluids flow problems. Moreover, mixed convection due to the gariotectic micro-organism helps to reduce the chances of sedimentation of nanoparticles in the base fluid. We would like to investigate an Exploration of Mixed Convection Casson Viscoelastic fluid Motion over a Complex Formation with CattaneoChristove Heat Flux.

Literature Review

A nano fluid is one that contains nanometer-sized particles known as nanoparticles. Nanofluids embrace metal alloys, halides, abrasive particles, or carbon nanofibers Because of their enhanced thermal properties, nanofluids are mainly used as coolants in heat exchangers such as heat pumps, electronic coolers (such as flat plate), and heating coils. TazaGul et al. [1] discussed study of hybrid nanofluid is quite important in several fields of science and engineering. Oauf et al. [2] investigated the Mhd boundary layer flow over an erodible stretched surface under the heat flux effect and got the exact solution. Choi et al. [3] demonstrated the improvement of nanotechnology by introducing a new fluid known as nanofluid. Among those who have contributed to this work are Mehdi et al. [4]. They proved that nanofluids have greater thermal transmittance and performance for temperature distribution than simple or core fluids used in sophisticated machines. The use of nanofluids in a variety of devices has showed strong heat transfer prospects. The effect of electron mobility on nanoparticle thermophysical properties. Nadeem et al. [5] investigated the role of hybrid nanofluid in heat enhancement rate in the published papers.

A heater with a passive heat sink up into the air is an indication of this. But since hot air rises, the air forced upwards the grill aids in temperature distribution. When both of these convection mechanisms work together to transfer heat, the result is mixed

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convection, which is defined as a combination of forced and natural convection. Because of their applications in science and engineering, the numerical results of mixed convection fluid flow bounded by an extending/compressing sheet have been published [6-9]. Ibrahim and Anbessa [10] used the spectral relaxation method (SRM) to investigate the mixed convection stream of a Maxwell nanofluid over a growing surface in a penetrable medium and discovered that an increase in Deborah number decreases both the stream and transverse velocity profiles, whereas an increase in the mixed convection parameter results in a backwards design.

Saravana and colleagues [11] investigated magnetohydrodynamic Casson fluids with variable thickness in an aligned magnetic domain. Atlas et al. [12] investigated the entropy of Casson nano fluids using the Cattaneo-Christov energy and mass flux design. The MHD Casson nano-fluid liquids with entropy generation was presented by Zubair et al. [13]. Dash et al. [14] used the Casson fluid model to investigate the flowing behavior of a homogeneous porous medium inside a pipe. The Casson fluid model, developed by Kumari et al. [15], is a preferred rheological model for many fluids, including blood and chocolate.

The Cattaneo-Christove heat flux model is used to describe viscoelastic heat transfer caused by an exponentially stretching sheet. The main points of this study can be summarized as follows: When compared to viscous fluid, the hydrodynamic boundary layer in viscoelastic fluid is thinner. Hayat et al. [16] investigated the impact of heat flux. Han et al. [17] used Cattaneo-Christov heat flux model to investigate the slip flow and heat transfer of Maxwell fluid bounded by a stretching sheet. Junaid Ahmad Khan et al. [18] discovered that fluid temperature has an inverse relationship with thermal relaxation time by employing a mathematical model. Ibrahim et al. [19] represent the convective flow of Eyring–Powell nano-fluid by using Cattaneo–Christov model. The effect of Cattaneo–Christov heat flux in the stagnation point fluid flow over a nonlinear stretching sheet with fluctuating thickness is described by Hayat et al. [20].

Chaim [21] and Abel et al. [22] studied an electrically conducting fluid over a linearly stretched sheet using viscous and joules dissipation. Bandar Bin-Mohsin [23] investigated how buoyancy responds when tested in MHD transport of Nanofluid by analyzing ohmic heating and viscous dissipation in his research. Barik et al. [24] investigated how viscous dissipation affects MHD flow in the presence of power law heat flux. Khadijah [25] determined the MHD flow and investigated gradient heat transport phenomena. Many researchers have also discussed the effect of viscous dissipation on Newtonian fluid flows. Murugesan et al. [26] reported the results of a thermo-solutal stratified dissipative MHD nanofluid flow. Sekhar et al. [27] demonstrated the multiple slip effects when fluid flow in a porous medium was considered. Makinde et al. [28] numerically illuminated the issues of MHD flows over a variety of surfaces, including vertical plates and stretching surfaces. Uddin [29] also explained fluid flow over a stretching porous surface in a Porous Medium. [30]. B. K. Swain et al. investigated the effect of viscous dissipation and joule heating on MHD flow and heat transfer past a stretching sheet embedded in a porous medium.[31] Mukhopadhyay described MHD boundary layer flow and heat transfer in a thermally stratified medium embedded in an exponentially stretching sheet.

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The Governing Equations

The following are the continuity and the **Navier Stokes equations**:

Continuity Equation:

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho\ \underline{V}) = 0$$

Momentum Equation:

$$\underline{f} - \nabla\pi + \mu\nabla^2\underline{V} = \rho\left(\frac{\partial \underline{V}}{\partial t} + (\nabla.\underline{V})\ \underline{V}\right)$$

Energy Equation:

$$\rho C_p\left(\frac{\partial T}{\partial t} + (\underline{V}\cdot\nabla)\,T\right) = K\nabla^2 T\ + \phi,$$

Where μ Viscosity Coefficient

$\underline{V}$ is the Speed Vector.

ρ is the Density.

$\underline{f}$ is the Body Force and π is the Pressure.

The Fluid Temperature is T whereas ϕ, C_p and K represents the Dissipation Function, Specific Heat and Heat Conductivity respectively.

The above equations become:

$$\nabla\cdot\underline{V}\ = 0,$$

$$\rho(\underline{V}\cdot\nabla)\underline{V} = \underline{f} - \nabla\pi + \mu\nabla^2\,\underline{V},$$

$$\rho C_p(\underline{V}\cdot\nabla)T = K\nabla^2\,T.$$

The following are the Equations for **Casson Nano-fluid Fluids**:

Continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla\cdot\left(\rho\ \underline{V}\right) = 0$$

Momentum Equation:

$$\frac{1}{\rho}\underline{f} - \frac{1}{\rho}\nabla\pi + \left(1+\frac{1}{\beta}\right)\mu\nabla^2\underline{V} = \frac{\partial \underline{V}}{\partial t} + (\nabla.\underline{V})\ \underline{V}$$

Energy Equation:

$$\rho C_p\left(\frac{\partial T}{\partial t} + (\underline{V}\cdot\nabla)\,T\right) = K\nabla^2 T\ + \phi + \tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right]$$

Where β is Casson Fluid Parameter.

τ is the Ratio of Effective Heat Capacity of Nano-particles and heat capacity of base fluid.

D_B is the Brownian Diffusion Coefficient.

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D_T is the Thermophoretic Diffusion Coefficient.

C is Concentration and T_∞ is Temperature away from surface.

The above equations become:

$$\nabla \cdot \underline{V} = 0,$$

$$\frac{1}{\rho}\underline{f} - \frac{1}{\rho}\nabla\pi + \left(1+\frac{1}{\beta}\right)\mu\nabla^2\underline{V} = (\nabla.\underline{V})\,\underline{V}$$

$$\rho C_p(\underline{V}\cdot\nabla)\,T = K\nabla^2 T + \tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right]$$

Numerical Procedure

When we use the Taylor series technique to analyze the nonlinear system, we could see that tackling the greater order differential equation takes a long time. There is a method known as the **Runge-Kutta method** that is used to solve such problems. C. Runge used it for the first time in 1894. W. Kutta extended and improved it after seven years in 1901. It has 5th order accuracy and is simple to compute using a device. The working rule for R-K Method is explained below with steps in detail.

Here, $\frac{dq}{dt_R} = f(t_R, q)$ at $q\left(t_{R_0}\right) = q_0$

Step I: To compute theq_1, we calculate successively

$$a_1 = h_R\, f(t_{R_0}, q_0)$$

$$a_2 = h_R\, f(t_{R_0} + \frac{h_R}{2}, q_0 + \frac{a_1}{2})$$

$$a_3 = h_R\, f(t_{R_0} + \frac{h_R}{2}, q_0 + \frac{a_2}{2})$$

$$a_4 = h_R\, f(t_{R_0} + h_R, q_0 + a_3)$$

Where h_R is step size

Step II: $a = \frac{1}{6}(a_1 + 2a_2 + 2a_3 + a_4)$

$q_1 = q_0 + a$ *and* $t_{R_1} = t_{R_0} + h_R$

Repeat his procedure to find the values of

q_1, q_2, q_3, q_4 *for* $t_{R_1}, t_{R_2}, t_{R_3}, t_{R_4}$ Etc.

The Runge-Kutta Method is used to solve simultaneous first-order equations. The simultaneous equation is presupposed as

$\frac{dq}{dp} = f_1(p,q,r)$; $\frac{dr}{dp} = f_2(p,q,r)$ with the initial condition $q(p_0) = p_0$

and $r(p_0) = r_0$, first we start (p_0, q_0, r_0) the increment of a and b respectively in q, r are given as:

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$$a_1 = h_R f_1(p_0, q_0, r_0) \; ; \; b_1 = h_R f_2(p_0, q_0, r_0)$$

$$a_2 = h_R f_1(p_0 + \frac{h_R}{2}, q_0 + \frac{a_1}{2}, r_0 + \frac{b_1}{2}) \; ;$$

$$b_2 = h_R f_2(p_0 + \frac{h_R}{2}, q_0 + \frac{a_1}{2}, r_0 + \frac{b_1}{2})$$

$$a_3 = h_R f_1(p_0 + \frac{h_R}{2}, q_0 + \frac{a_2}{2}, r_0 + \frac{b_2}{2}) \; ;$$

$$b_3 = h_R f_2(p_0 + \frac{h_R}{2}, q_0 + \frac{a_2}{2}, r_0 + \frac{b_2}{2})$$

$$a_4 = h_R f_1(p_0 + h_R, q_0 + a_3, r_0 + b_3) \; ;$$

$$b_4 = h_R f_2(p_0 + h_R, q_0 + a_3, r_0 + b_3)$$

$$a = \frac{1}{6}\left(a_1 + 2a_2 + 2a_3 + a_4\right)$$

$$b = \frac{1}{6}\left(b_1 + 2b_2 + 2b_3 + b_4\right)$$

Hence, $q_1 = q_0 + a$, $r_1 = r_0 + b$

Compute the number of q_2, r_2 replace p_0, q_0, r_0 by p_1, q_1, r_1

The Runge-Kutta method is used to solve second order differential equations,

Taking the second order differential equation into account,

$$\frac{d^2q}{dp^2} = \psi\left(p, q, \frac{dq}{dp}\right) \; ; \; q(p_0) = q_0 \; ; \; q'(p_0) = q_0'$$

Let $\frac{dq}{dp} = w$ then $\frac{d^2q}{dp^2} = \frac{dw}{dp}$

Put these values in the above equation then we get,

$$\frac{dw}{dp} = \psi(p, q, r) \; ; q(p_0) = q_0 \; ; r(p_0) = r_0$$

The problem reduced to solving the simultaneous equation

$\frac{dq}{dp} = w = f_1(p, q, r)$ *and* $\frac{dw}{dp} = w' = f_2(p, q, r)$ subject to the condition

$q(p_0) = q_0 \; ; w(p_0) = w_0$

Then overcome as discussed in the previous section.

Mixed Convection Casson Nano-Fluid Motion Over An Extending Sheet With Cattaneo_Christov Heat Flux

In this Section, we presented physical problem and mathematical formulation of the previous work by Swain [30]. Physical interpretation and mathematical analysis was made for ‘‘Viscous Dissipation and Joule Heating Effect On MHD Flow And Heat Transfer Passed a Stretching Sheet in a Porous Medium’’. Joule heating and viscous dissipation are important physical characteristics for heat transportation in viscous fluids flow [13] and [27]. The governing nonlinear partial differential equations are reconstituted as ordinary differential equations then the transformed equations were

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Print ISSN 3007-3189
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solved numerically by Runge Kutta Method.
We studied this problem and observed its limitation to the viscous fluid flow only. Moreover, heat transportation is sluggish because of small thermal conductivity of the base fluid. However, the physical and mathematical description of the problem is given below:

Mathematical Analysis:
Consider the movement of an inviscid Newtonian fluid across a stretching or shrinking sheet. The source would be at the fluid's slit medium through which the sheet is drawn. The x-axis runs parallel to the sheet, whereas the y-axis runs perpendicular. The outer layer has been assumed to be uniform as time passes with velocity $U = U_0 e^{x/L}$, Where U_0 and L are the reference velocity and the length respectively. The Flow geometry and coordinate system is explained in figure 3.1.1. The mass suction velocity is v_w. The sheet's layer is maintained at constant heat transfer q_w and the ambient fluid temperature must be T_∞.A magnetic field that exists outside the body $\overrightarrow{B_0}$ is used adjacent to the y-axis. The mediated magnetism and the exterior electric potential are ignored.

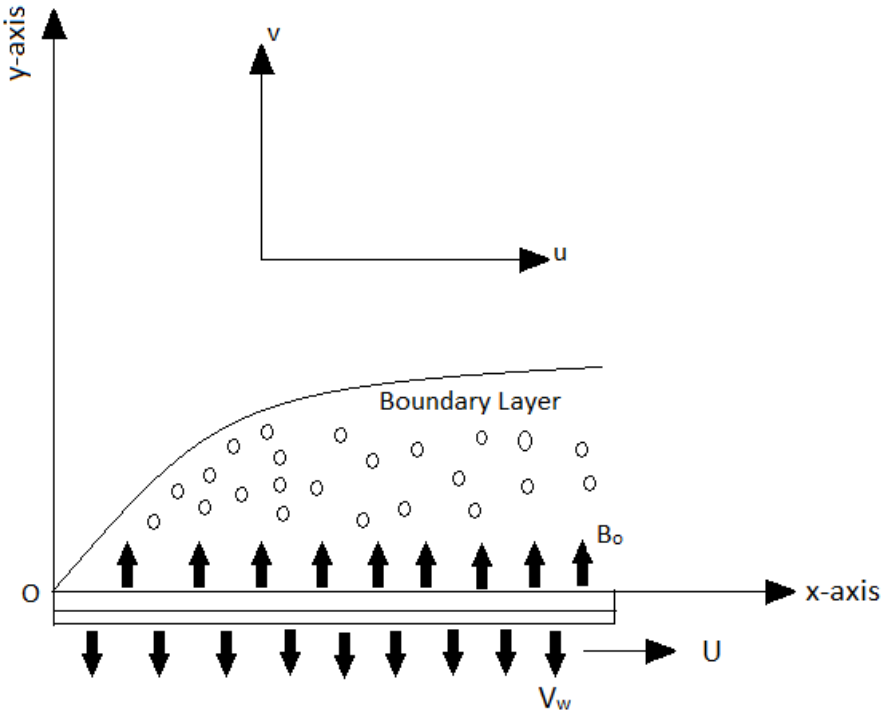


Fig. 3.1.1 Flow geometry and coordinate system

The governing equations of motion are,

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$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3.1.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma {B_0}^2}{\rho}u-\frac{\upsilon}{k_p^*}u \tag{3.1.2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+\frac{q'''}{\rho c_p}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma B_0^2}{\rho c_p}u^2 \tag{3.1.3}$$

The associated boundary conditions are

$$u=U,\ v=-v_w,-k\left(\frac{\partial T}{\partial y}\right)=q_w \ at\ y=0 \tag{3.1.4}$$

$$u\to 0,\ T\to T_\infty \ as\ y\to\infty$$

where v and u are components of velocity along y-axis and x-axis respectively, T, $T_w, \upsilon, \rho, k_p^*, \sigma, C_p, q_r, T_\infty, q_w$ are temperature, uniform temperature near the sheet, kinematic viscosity, density of fluid, permeability of porous medium, electrical conductivity, specific heat at constant pressure, radiative heat flux and temperature away from sheet respectively. Here it is assumed that , $v_w=v_0e^{x/2L}$ where v_0 is constant According to Rosseland Approximation [32], the

radiative heat flux q_r is described as $q_r=-\left(\frac{4\sigma^*}{3k^*}\right)\frac{\partial T^4}{\partial y}$

Now as reported in [33], assuming the meager diversity in temperature stream, expanding T^4 in a taylor series about T_∞ and neglecting higher order terms, We get

$T^4\cong 4T_\infty^3-3T_\infty^4$

Introducing dimensionless coordinates

$$\eta=y\sqrt{\frac{U}{2\upsilon L}},\ u=Uf'(\eta), v=-\sqrt{\frac{\upsilon U}{2L}}\left(f(\eta)+\eta f'(\eta)\right),\ \theta(\eta)=\frac{k}{q_w}\left(T-T_\infty\right)\sqrt{\frac{\mathrm{Re}}{2L^2}},$$

$$M=\frac{2\sigma B_0^2 L}{\rho U}, P_r=\frac{\mu C_p}{k}, R=\frac{16\sigma^* T_\infty^3}{3k^*k}, f_w=v_0\left(\frac{\mu U_0}{2\rho L}\right)^{-1/2}, k_p=\frac{k_p^* U}{2\upsilon L}, \mathrm{Re}=\frac{UL}{\upsilon},$$

$$E_c=\frac{U^2k}{C_pq_w}\sqrt{\frac{\mathrm{Re}}{2L^2}}, J=E_cM$$

And writing the internal heat generation or absorption q''' as (Chamkha and Khaled [34])

$$q'''=k\left(\frac{\mathrm{Re}}{2L^2}\right)\left[a^*\left(T_w-T_\infty\right)e^{-\eta}+b^*\left(T-T_\infty\right)\right]$$

The corresponding ordinary differential equations of the governing equations (3.1.1),(3.1.2) and (3.1.3) are

$$f'''+ff''-f'^2-(M+\frac{1}{k_p})f'=0 \tag{3.1.5}$$

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Print ISSN 3007-3189
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$$(1+R)\theta''+\mathrm{P_r}\, f\,\theta'+\mathrm{P_r}\, f'\theta+P_rE_c f''^2+P_rJf'^2+a^*e^{-\eta}+b^*\theta=0 \qquad (3.1.6)$$

The associated boundary condition becomes

$$f = f_w,\ \ f'=1,\ \ \theta'=-1 at\ \eta=0$$
$$f'\to 0,\ \theta\to 0 as\ \eta\to\infty \qquad (3.1.7)$$

Where, keeping in view, the limitations of the problem studied by Swain [30], we extended this work for non-newtotian base fluid and formulated the problem by utilizing Casson Model. It is becase the diffusion of nano-particles in base fluid helps to enhance its thermal conductivity and at the same time the behavior of base fluid is better to be exhibited by Non-Newtotnian dynamics. The other features of the extended work include mixed convection and Cattaneo-Christov heat flux.

Keeping in view, the short comings of the above mentioned previous, we extended the study. It is conceived to remodel the problem for non-Newtonian fluid by employing Casson formula. The thermal conductivity is improved by nano-particles diffusion. Non-Fourier heat diffusion is employed

The flow of Casson nano fluid is considered due to stretch horizontal sheet. The flow is assumed to be incompressible and time independent. The fluid is transported through Darcy porous matrix. A magnetic fluid of constant strength B_0 acts perpendicular to the flow direction. The sheet is permeable with mass suction velocity ($-v_w$). The wall temperature is T_w and concentration at wall is C_w. A dilute homogenous diffusion of nano-particles is made in the base liquid. The two slips conditions of Brownain motion and themopheris are taken into account the following Burjinioro Modoel. The non Fourier heat diffusion is presumed with Cattaneo-Christov theory. Joule heating effect and thermal dissipation are retained as in previous work.

Numerical Procedure

This segment encloses numerical outcomes from the non-dimensional nonlinearity accompanying ordinary differential equations (3.2.7) to (3.2.9) with boundary conditions (3.2.10) that are integrated utilizing the RK-4 strategy. To achieve this analytical technique, the differential equations (3.2.7) to (3.2.9) are incorporated into a first-order scheme by introducing new variables, as displayed below:

$$dy_1 = y_2$$

$$dy_2 = y_3$$

$$dy_3 = \frac{1}{\left(1+\frac{1}{\beta}\right)}\left[2y_2^2+yy-\left(M+\frac{1}{k_p}\right)y_2+\lambda y_4\right]$$

$$dy_4 = y_5$$

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$$dy_5 = (-1)\left[\Pr\left(y_2 y_4 + y_1 y_5\right) + \Pr Nb y_5 y_7 + \Pr Nt\, y_5^2 \right.$$

$$+ \Pr b_1 \left[\left(y_1 y_3 - \eta y_2 y_3 - y_2^2\right) y_4 + \left(y_1 y_3 + \eta y_2 y_3\right) y_5 + y_1^2 dy_5 \right]$$

$$dy_6 = y_7$$

$$dy_7 = (-1)\left[Le\left(y_2 y_6 + y_1 y_7\right) + \frac{Nb}{Nt} dy_5 \right]$$

Associated boundary conditions are

$$y_3 = a, y_5 = b, y_7 = c$$

$$y_1 = f_w, y_2 = 1, y_4 = 1, y_5 = -1, y_6 = 1 at\, \eta = 0$$

$$y_2 \to 0, y_4 \to 0, y_6 \to 0\, as\, \eta \to \infty$$

Results and Discussion

Computational code as described in the previous section 3.3 is run on Matlab platform. In order to explore the nature of flow (speed) f', fluid temperature θ and volume fraction of nano species, the influential variables/parameters are varied in substantial ranges. Before this, limited evaluation is carried out to find the results for $-f''(0)$ (local skin friction) to make a comparative sheet in the form of table 4.1.1. Here, the validity of the numerical scheme seemed to be convincing for its accuracy on basis of closed agreement in the two sets of values (previous [30] and present). By gaining confidence, the outputs for local skin friction ($-f''(0)$), local heat transfer rate $-\theta'(0)$ and Sherwood number $-\phi'(0)$ are computed extensively. The role of particular variables/parameters on the dependent physical quantities is enumerated with in notable ranges of that parameter and other are fixed as follow:

The change in $-f''(0)$ is demonstrated in table 4.1.1, where it seems that magnitude of $-f''(0)$ enhances directly with incremented inputs of the parameters/numbers M, f_w, k_p and β. Actually, these parameters are responsible to augment the resistance to the flow and hence skin friction factor becomes larger. However, decrement in $-f''(0)$ is seen in case of increased mixed convection which boots the flow velocity. Table 4.1.2 enlists the outputs for local heat transfer rate, which increases with the strength of $\Pr$ and b_1 but it is declined against Nb and Nt. It is also noticed that for constant wall temperature boundary, the heat temperature rate ($-\theta(0)$) and for heat flux boundary, heat transfer rate is ($\theta(0)$). It means that in first case, the temperature at the wall is higher than that of fluid but the situation is revered in case of heat flux. Table 4.1.3 presents the permuted values are $-\phi'(0)$ indirect response to Le and Nb. It is noticed that $-\phi'(0)$ is depreciated against Nt.

Figures 4.2.1 to 4.2.11 are drawn for the velocity $f'(\eta)$ of nano particles volume

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fraction $\phi(\eta)$ and temperature field $\theta(\eta)$ for Newtonian and non Newtonian fluid transportation over the stretching geometry. The strength of magnetic parameter is improved to see its impact on velocity $f'(\eta)$ as delineated in fig. 4.2.1. The velocity of flow decelerates notably for both the fluids (Newtonian and non Newtonian). Physically, the growing strength of M provides boost to the resistive force (Lorentz force) which is called to play through interaction of the electric and magnetic fields. Fig.4.2.2 displays the decreasing influence of porosity parameter k_p on $f'(\eta)$. The larger k_p means the lesser permeability of the porous medium. Hence a greater resistance is offered to the flow and the velocity is decreased. In Fig.4.2.3, the mass suction f_w imparts its role on $f'(\eta)$, the velocity curve of declines against the greater mass suctions through the wall of the sheet.

The larger inputs to the mixed convection parameter λ is responsible to promote the fluid velocity, as noticed from Fig.4.2.4. The incremented λ means larger buoyancy affect to enhance speed of flow. The number $\Pr$ is related reciprocally with thermal diffusivity. The lager values of $\Pr$ permits lesser heat diffusion in the fluid and hence reduction in fluid temperature is resulted (see Fig. 4.2.5). Similarly, Fig.4.2.6 indicates the reduction in temperature $\theta(\eta)$ against Cattaneo-Christove parameter b_1. However, Fig.4.2.7 and Fig.4.2.8 demonstrate rising of the fluid temperature $\theta(\eta)$ when Brownian motion parameter Nb and the thermophoretic parameter Nt are strengthened. The higher values of Nb means enhanced random motion of nano-entities which establishes improvement in the heat diffusion and the fluid temperature rises. Thermophoretic effect as signified by Nt is responsible for the movement of nano-particles from hotter regime to the colder one. In this way, heat diffuses in the fluid to raise $\theta(\eta)$.

From Fig.4.2.9, It is observed that the volume fraction of nano-particles $\phi(\eta)$ reduces significantly when Lewis number Le is boosted. Lewis number is inversely related to the coefficient mass diffusivity. Hence, larger Le means lesser mass diffusion of nano-paticles which results in lower concentration. In the same manner, the volume fraction of nano-particles $\phi(\eta)$ recedes against Nb as depicted in Fig.4.2.10. The faster random movement of the nano-particles diffuses rapidity in the fluid and their concentration is reduced. Fig.4.2.11 portraits the increasing behavior of $\phi(\eta)$ as direct response to Nt. The therophoretic parameter Nt signifies the movement of the nano particles towards colder fluids and diffusion of the particle is improved to enhance $\phi(\eta)$

Fig. 4.2.12 to fig. 4.2.15 are established to sketch the temperature distribution for heat flux boundary conditions. Temperature distribution $\theta(\eta)$ enhances directly with the increments in Nt and Pr but it is declined against Nb and b_1.

Tables

Table 4.1.1: Values of skin friction coefficient $-f''(0)$ by varying each of these M, f_w, k_p, β and λ

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M	f_w	k_p	β	λ	$-f''(0)$
0	1.5	1	0.5	0.1	3.6020
0.5					3.8563
1.0					4.0904
1.0	0.5				3.5188
	1.0				3.7953
	1.5				4.0904
	1.5	0.1			3.6548
		0.5			3.8563
		1.0			4.0904
		1.0	0.1		7.0318
			0.3		4.7391
			0.5		4.0904
			0.5	0.1	4.0704
				0.2	4.0546
				0.3	4.0186

Table 4.1.2: Values of local heat rate by varying each of these $\Pr$, Nb, Nt and b_1

Pr	Nb	Nt	b_1	$-\theta'(0)$ Wall temp.	$\theta'(0)$ Heat Flux
0.7	0.1	0.1	0.5	1.3428	1.4044
1.0				1.6107	1.7130
1.2				1.7510	1.8814
1.0	0.1			1.6107	1.7130
	0.2			1.5020	1.5965
	0.3			1.4005	1.4878
	0.1	0.1		1.6107	1.7130
		0.2		1.5634	1.7685
		0.3		1.5185	1.8271
		0.1	0.1	1.8063	1.9372
			0.3	1.7044	1.8202
			0.5	1.6107	1.7130

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Table 4.1.3: The values Sherwood no. $-\phi'(0)$ for various values of Le, Nt and Nb .

Le	Nt	Nb	$-\phi'(0)$
4	0.1	0.1	5.7661
5			7.3414
6			8.8983
4	0.1		5.7661
	0.2		4.6787
	0.3		3.6670
	0.1	0.1	5.7661
		0.2	6.4030
		0.3	6.6130

Table 4.1.4: Comparison with earlier results

Parameters	Swain[30]	Kadijah[25]	Present Results
	$f''(0)$	$f''(0)$	$f''(0)$
$M = 0$	–2.203913	–2.20348	–2.20348
$M = 1.5$	–2.658565	–2.65822	–2.65822
$f_w = 1$	–2.028286	–2.02809	–2.02809
$f_w = 2$	–2.745943	–2.74565	–2.74565

Figures

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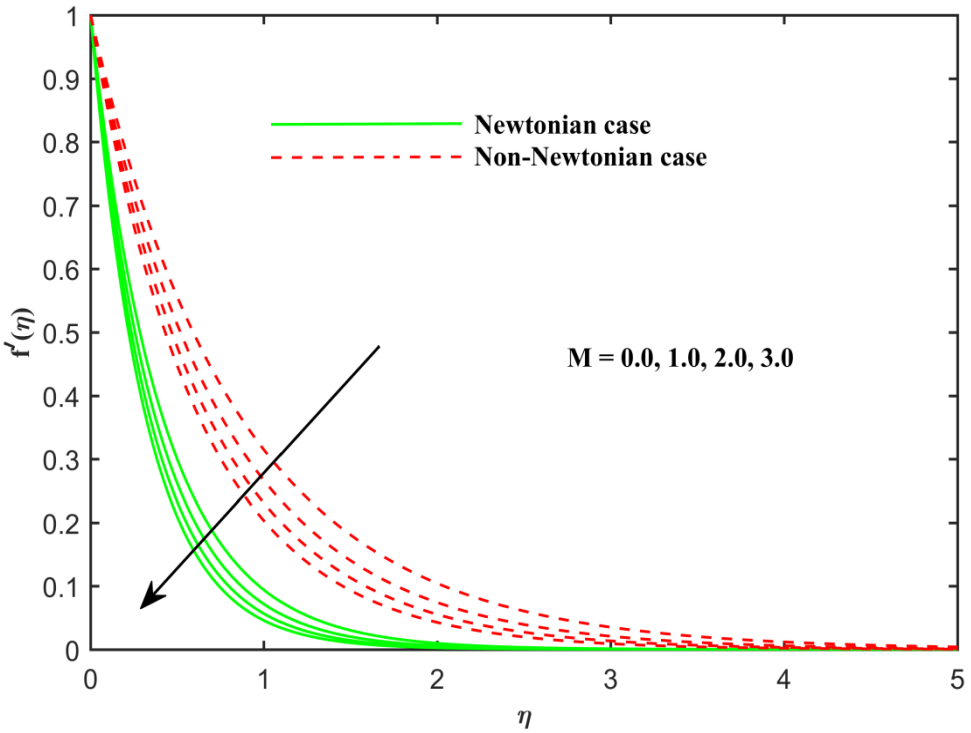


Figure: 4.2.1 Fluctuation of $f'(\eta)$ along with M

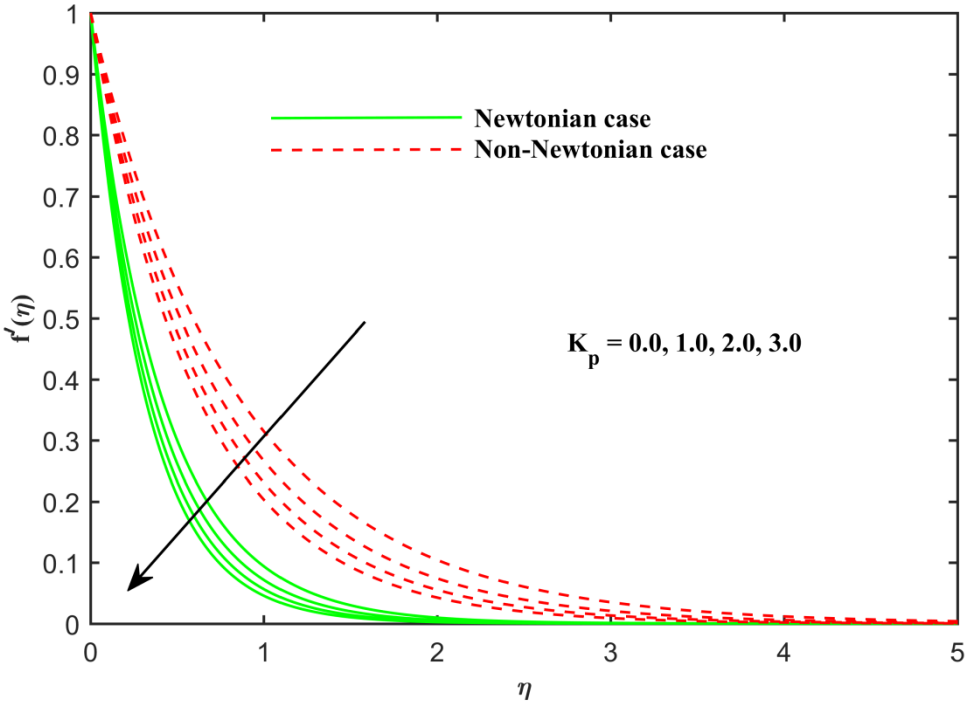


Figure: 4.2.2 Fluctuation of $f'(\eta)$ along with k_p

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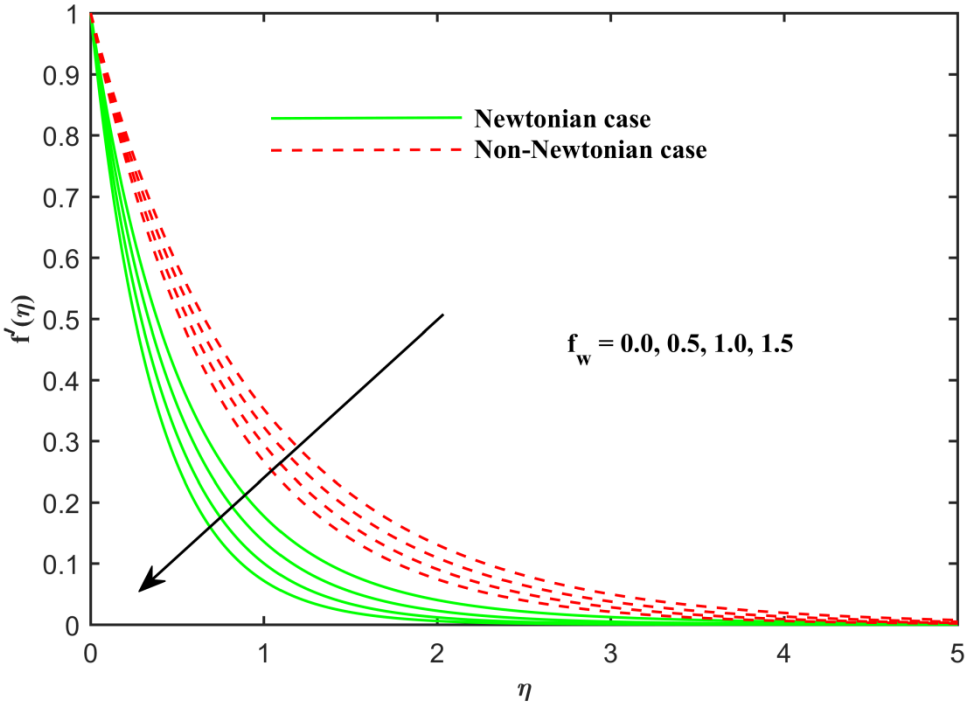


Figure: 4.2.3 Fluctuation of $f'(\eta)$ along with f_w

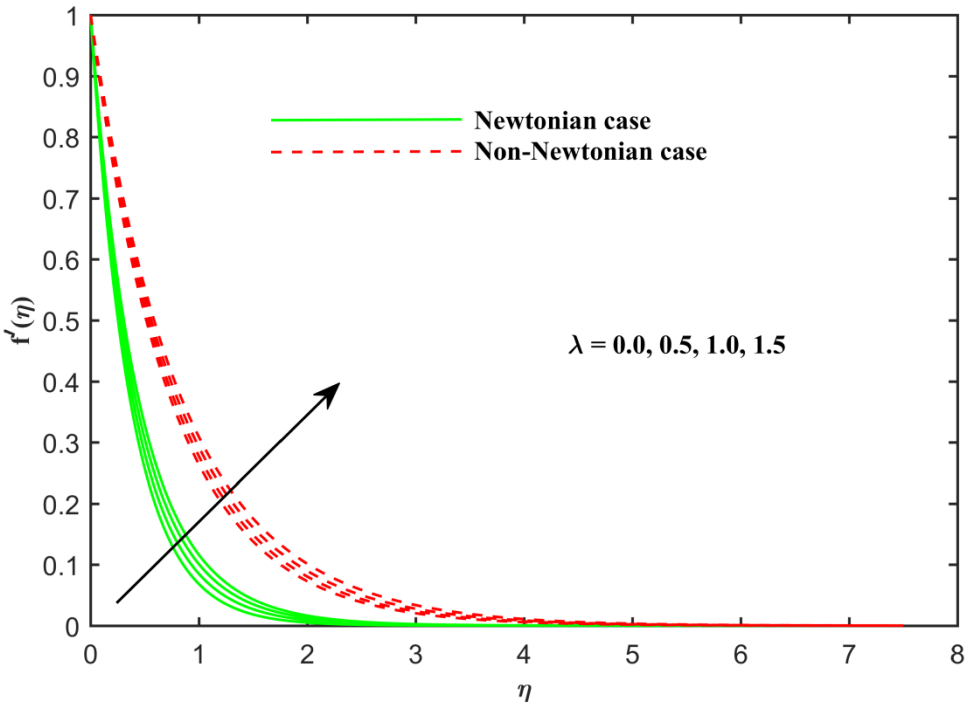


Figure: 4.2.4 Fluctuation of $f'(\eta)$ along with λ

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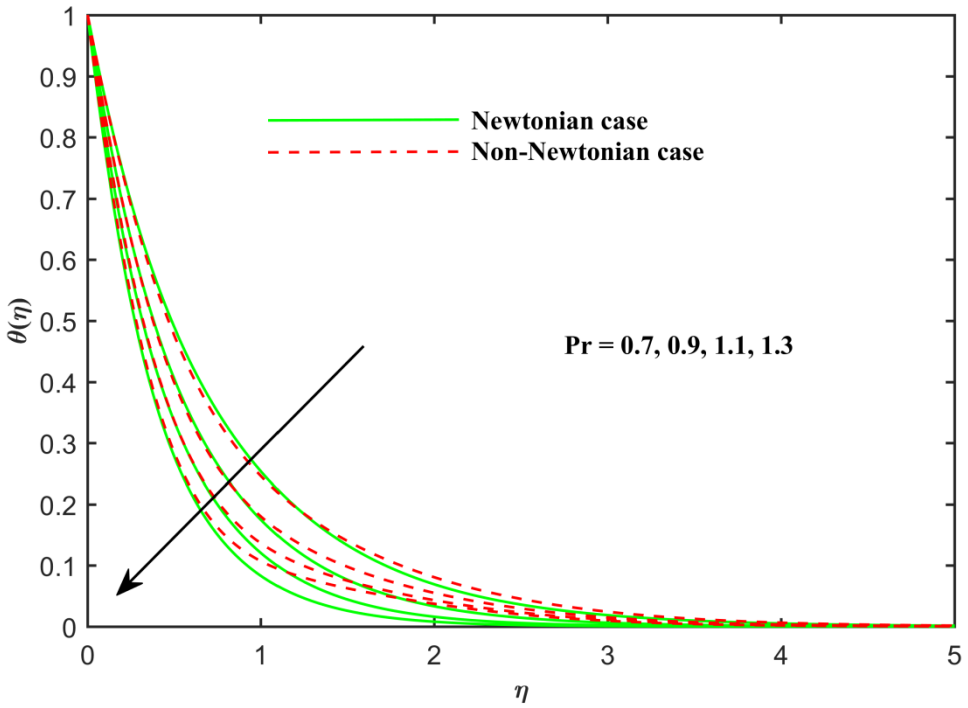


Figure: 4.2.5 Fluctuation of $\theta(\eta)$ along with Pr

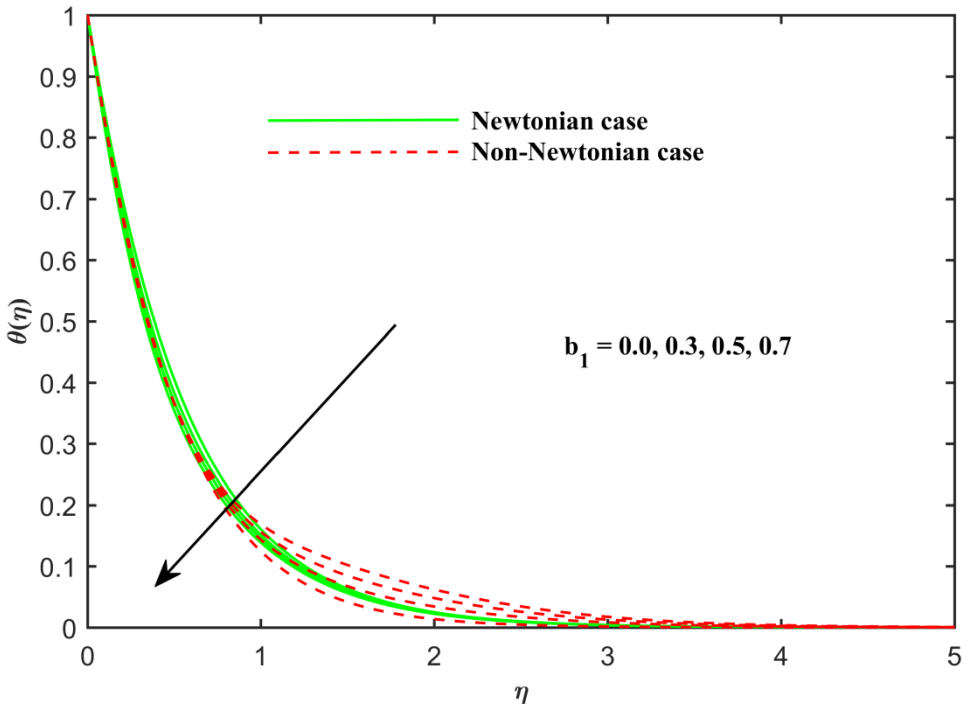


Figure: 4.2.6 Fluctuation of $\theta(\eta)$ along with b_1

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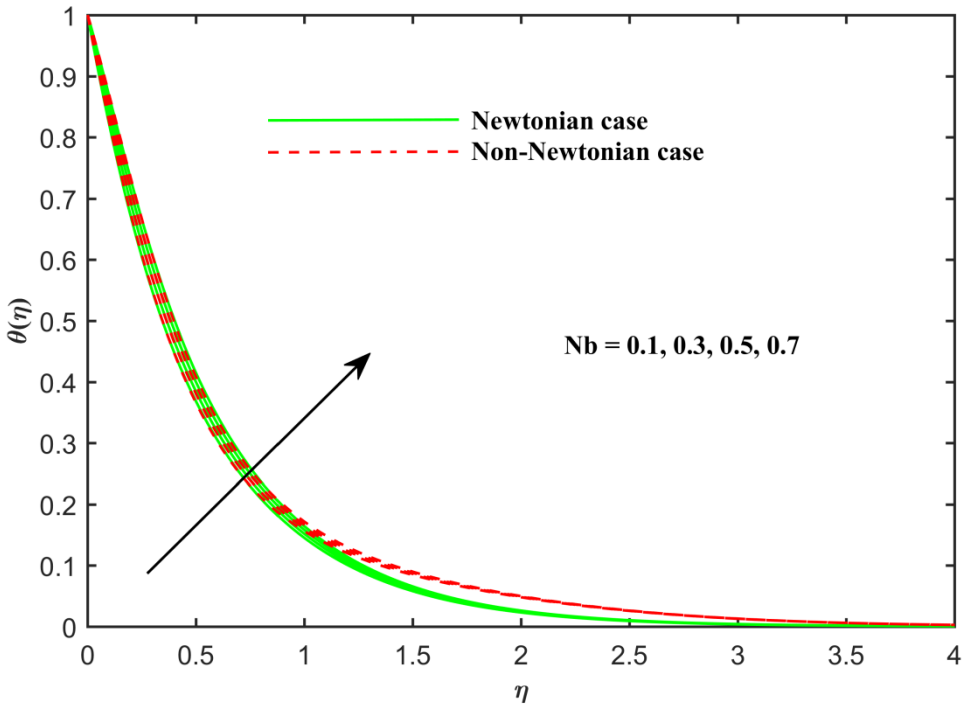


Figure: 4.2.7 Fluctuation of $\theta(\eta)$ along with Nb

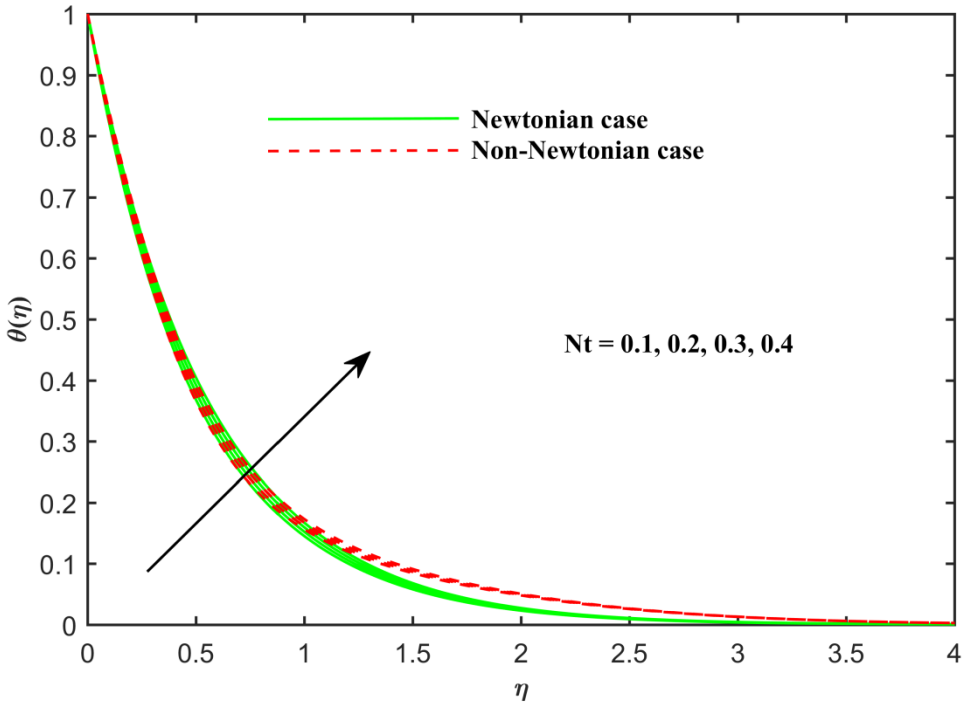


Figure: 4.2.8 Fluctuation of $\theta(\eta)$ along with Nt

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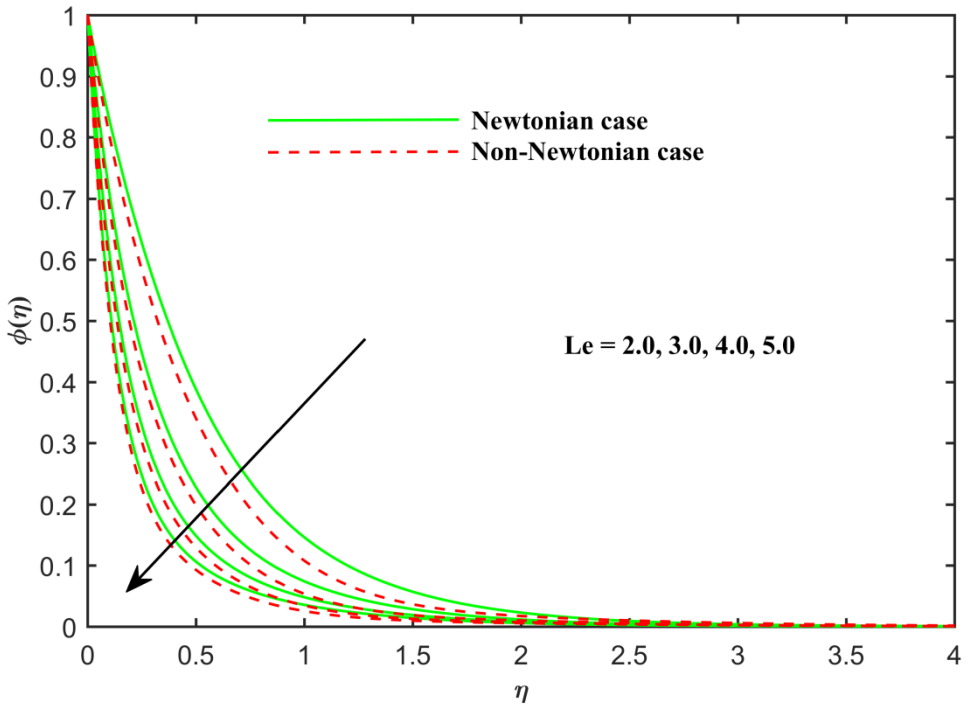


Figure: 4.2.9 Fluctuation of $\phi(\eta)$ along with Le

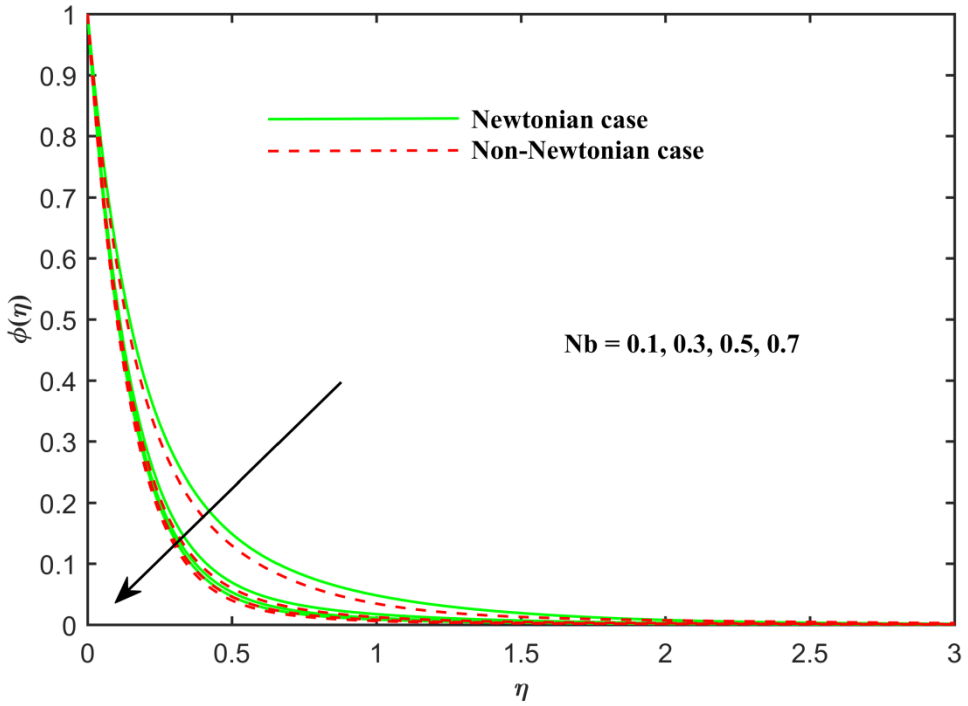


Figure: 4.2.10 Fluctuation of $\phi(\eta)$ along with Nb

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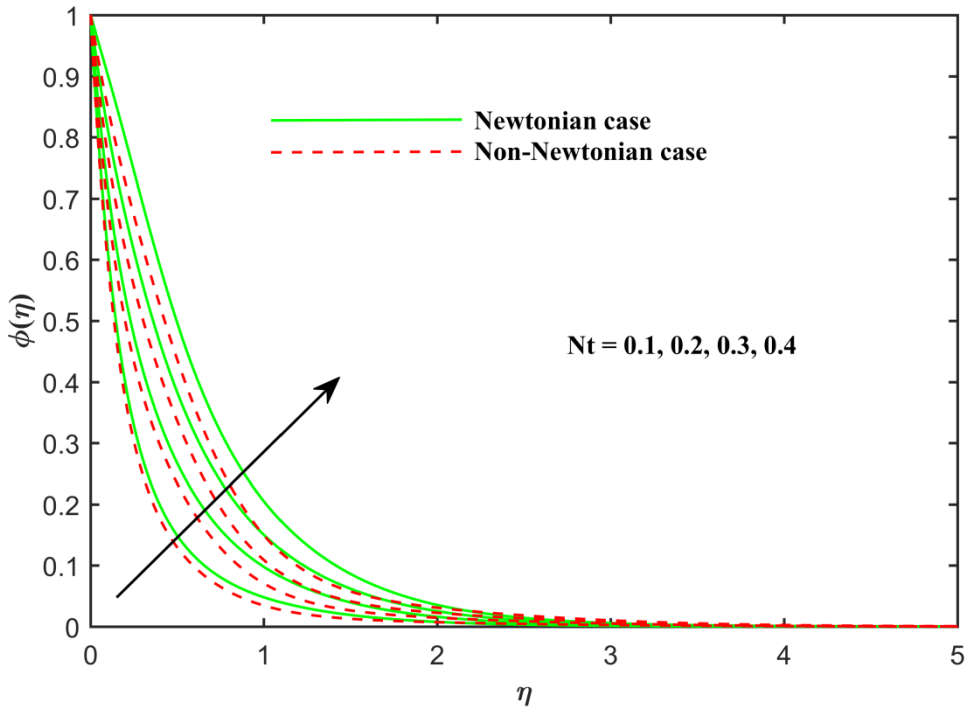


Figure: 4.2.11 Fluctuation of $\phi(\eta)$ along with Nt

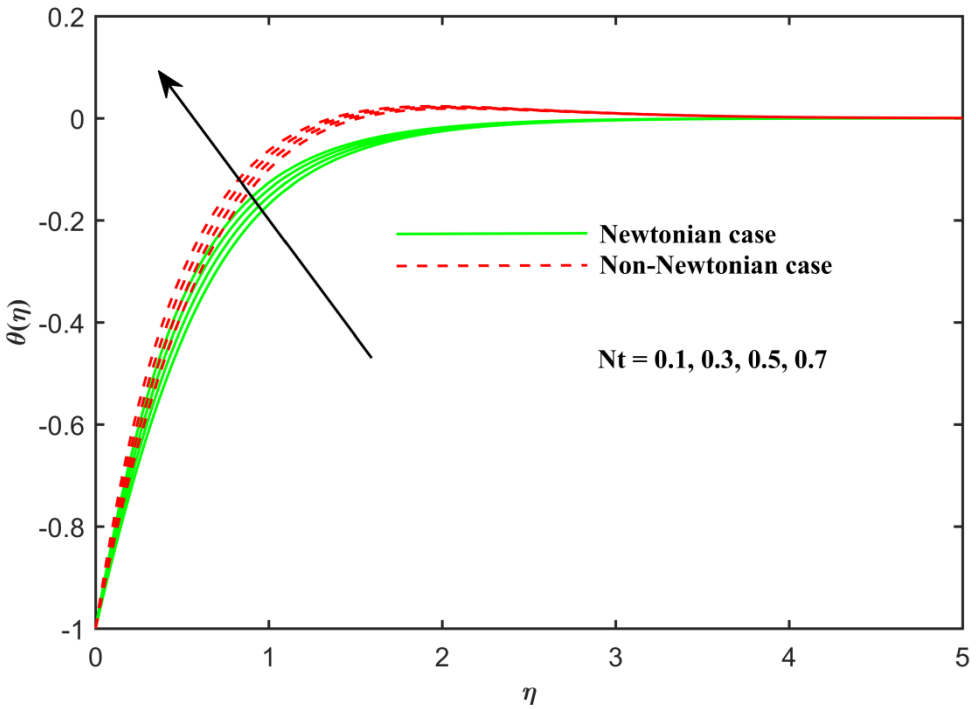


Figure: 4.2.12 Fluctuation of $\theta(\eta)$ along with Nt

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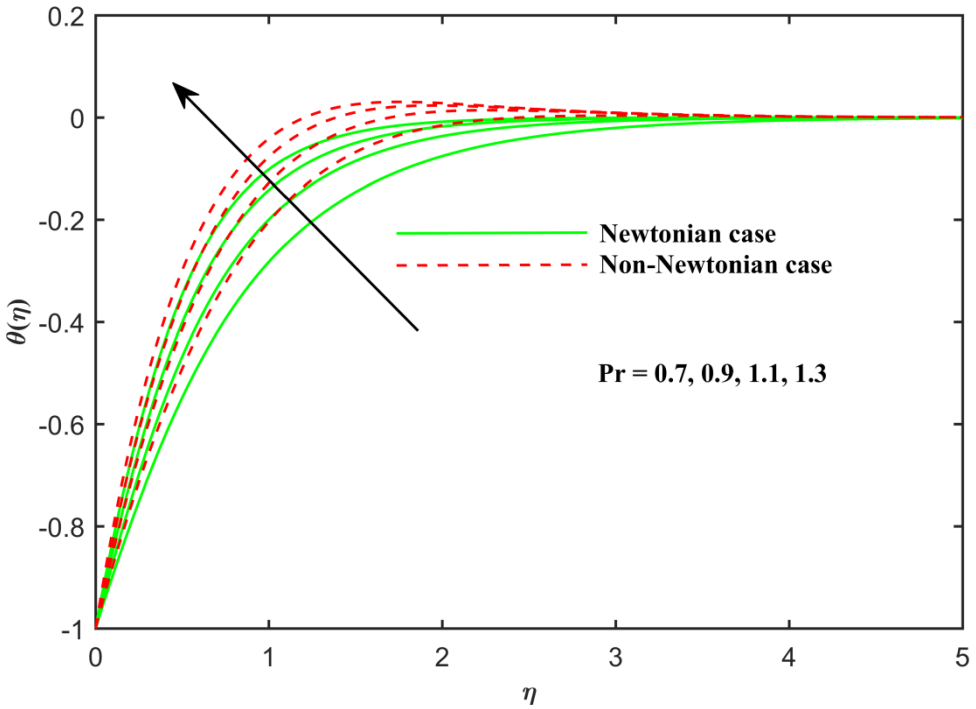


Figure: 4.2.13 Fluctuation of $\theta(\eta)$ along with Pr

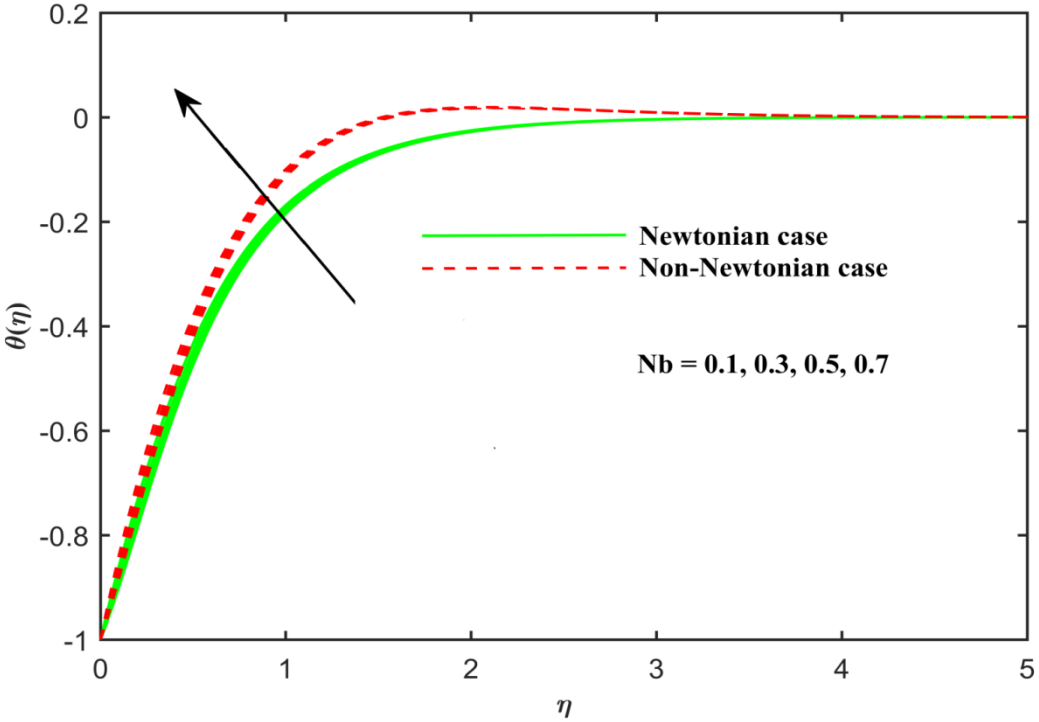


Figure: 4.2.14 Fluctuation of $\theta(\eta)$ along with Nb

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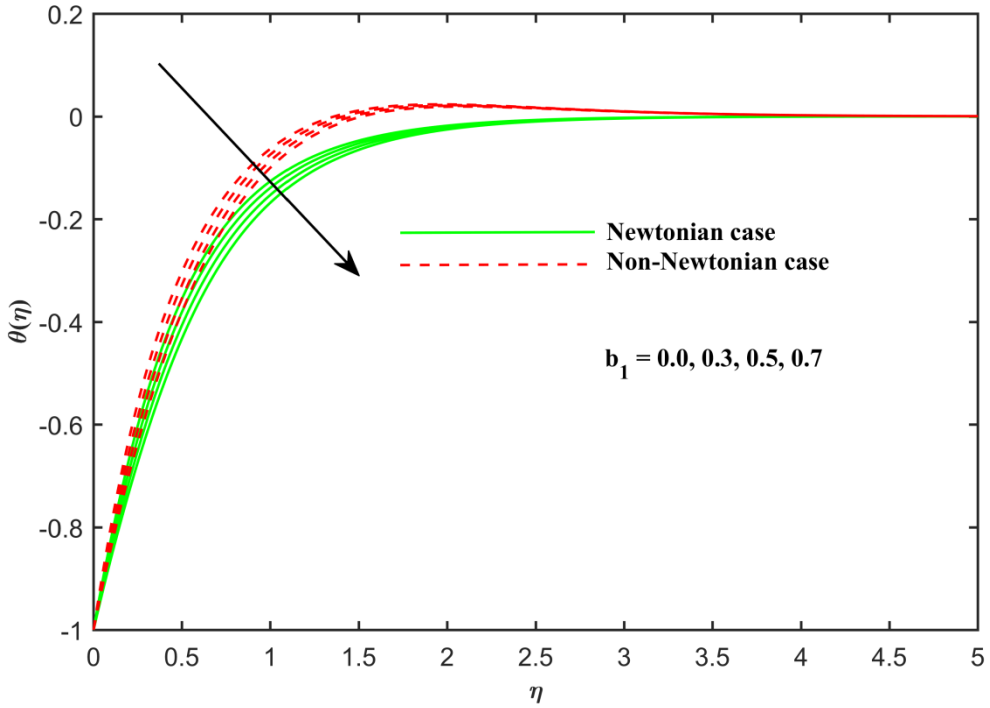


Figure: 4.2.15 Fluctuation of $\theta(\eta)$ along with b_1

Conclusion

In the pursuit for increment in heat distribution, we devised a theoretical study on MHD flow of Casson Nano-fluids across a porous matrix due an extending porous sheet. The addition attribute include Joule heating and thermal dissipation. The leading equation in this transformed ordinary differential structure are solved numerically by hiring Runge-Kutta Method and shooting tool to be coded in Matlab script. The outputs are evaluated through enumeration of the variation of physical quantities as influenced by the pertinent parameters. The mentioned outcomes are briefly described as below:

The magnitude of $-f''(0)$ enhances directly with incremented inputs of the parameters M, f_w, k_p and β because of exceeding resistance to the flow but $-f''(0)$ is recedes against mixed convection.

The Nusselt number $-\theta'(0)$ grows with Pr and b_1 but it decreases reciprocally with Nb and Nt.

Sherwood number $-\phi'(0)$ is directly rising with Le and Nb but it is depreciated against Nt.

The flow velocity $f'(\eta)$ decelerates due to strength of M, mass suction f_w and porosity parameter k_p

The mixed convection parameter λ improves $f'(\eta)$.

The larger P_r and b_1 reduces temperature $\theta(\eta)$.

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The temperature $\theta(\eta)$ rises due to Nb and Nt.

The volume fraction of nano-particles $\phi(\eta)$ reduces Le and Nt are boosted but $\phi(\eta)$ is incremented with Nt.

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