

A Study of Maxwell Ternary Hybrid Nanofluid (BN TiO_2 – Al_2O_3) with Blood as Carrier for Enhanced Thermal Transport in Biomedical Porous Systems under Magnetic Field and Suction

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ABSTRACT

The research focuses on the parameters of flow and heat transfer of a Maxwell ternary hybrid nanofluid containing BN, TiO_2 , and Al_2O_3 nanoparticles suspended in blood. The evaluation is done on an extended surface that is inside a porous biological matrix. The ternary hybrid nanofluid is taken as non-Newtonian Maxwell fluid in order to reflect the viscoelastic nature of blood, which plays an important role in biomedical applications. Its model incorporates the influence of transverse magnetic field (magnetohydrodynamics or MHD), suction, and heat radiation, conditions that produce realistic conditions of hyperthermia as an alternative cancer treatment method and of targeted drug delivery. Nanoparticles of BN, TiO_2 and Al_2O_3 are very thermally conductive and biocompatible and thus their use would enhance heat conduction in the bio-thermal systems. Through similarity transformations, the governing partial differential equations are simplified into nonlinear ordinary ones which can be solved numerically via the Numerical solution of Boundary value problems 11 built in MATLAB named as bvp4c. This paper tallies significance of significant dimensionless quantities like Maxwell relaxation time, magnetic parameter and suction parameter, nanoparticle volume fractions and the permeability of a porous medium. The findings demonstrate that when the concentration of the nanoparticle triplet is augmented, the thermal conductivity and temperature distribution of the hybrid nanofluid also enhances largely. Application of a magnetic field would result into Lorentz forces, which suppress the flow speed and enhance heating along the boundary layer. The surface suction contributes to the sustenance of the thermal boundary layer which causes the controlled and localized conveyance of heat. In addition, viscoelastic nature of the Maxwell fluid influences the flow resistance which could be manipulated in order to achieve maximum heat handling within the biomedical systems. The comprehensive study is devoted to synergistic effects of ternary nanoparticles, non Newtonian fluid behaviour, and magnetothermal dynamics. The results hold an interesting meaning to the creation and setup of enhanced nanofluid based biomedical devices on tumor hyperthermia, focused drug delivery systems, and perfect chilled systems in clinical practice.

Keywords: *Maxwell ternary hybrid nanofluid, BN $\text{TiO}_2\text{-Al}_2\text{O}_3$ nanoparticles, biomedical porous systems, magnetic field effects, blood-based coolant, thermal transport enhancement*

1. Introduction

The rapid development of **nanotechnology** has significantly influenced the field of fluid mechanics, particularly in the enhancement of heat transfer and thermal management applications. Among these innovations, **nanofluids**—fluids engineered by suspending nanoparticles in conventional base fluids—have emerged as a promising solution due to their superior thermophysical properties, including enhanced thermal conductivity, heat capacity, and convective heat transfer rates. Traditional base fluids such as water, ethylene glycol, or blood exhibit limitations in heat transport, which can restrict their performance in advanced biomedical and industrial applications. The integration of nanoparticles into these fluids overcomes these limitations, enabling **higher efficiency in thermal systems**.

A notable advancement in this field is the development of **hybrid nanofluids**, particularly **ternary hybrid nanofluids (THNFs)**, where three different nanoparticles are dispersed in a base fluid. The combination of multiple nanoparticles produces a **synergistic effect**, enhancing the fluid's overall thermal properties beyond what is achievable with single or binary nanoparticle suspensions. This research focuses on **BN- $\text{TiO}_2\text{-Al}_2\text{O}_3$ /blood nanofluids**, selected due to their complementary properties:

- **Boron Nitride (BN):** High thermal conductivity, biocompatibility, and electrical insulation make BN suitable for safe biomedical applications, including implantable devices and drug delivery.
- **Titanium Dioxide (TiO_2):** Offers chemical and photothermal stability, making it ideal for applications involving radiation exposure or long-term use.
- **Aluminum Oxide (Al_2O_3):** Economical, chemically stable, and thermally conductive, enhancing fluid durability and heat transfer efficiency.

When suspended in **blood**, these nanoparticles create a **biocompatible ternary hybrid nanofluid** capable of improving several biomedical processes, such as **hyperthermia treatment, targeted drug delivery, and thermal management of artificial organs**. Blood, with its **non-Newtonian, shear-thinning, and viscoelastic behavior**, adds complexity to the fluid dynamics, requiring advanced modeling approaches. The **Maxwell fluid model** is employed to capture these viscoelastic properties, integrating both elastic and viscous components to realistically simulate physiological flows.

The interaction of such nanofluids with **magnetohydrodynamics (MHD)**, **wall suction/injection**, and **porous media** is critical in biomedical applications. Magnetic fields influence the motion of electrically conducting nanofluids, enabling **targeted drug delivery, controlled blood flow, and localized heat management**, while porous structures mimic tissues and vascular networks, affecting nanoparticle diffusion and heat transport. Wall suction and injection mechanisms simulate real medical interventions, such as fluid removal, drug administration, or irrigation, providing a more accurate representation of clinical scenarios.

Despite the extensive research on **conventional and binary hybrid nanofluids**, studies on ternary hybrid nanofluids in **porous biomedical domains remain limited**, particularly when considering the combined effects of non-

Newtonian behavior, magnetic fields, and synergistic nanoparticle interactions. Most existing models oversimplify blood as a Newtonian fluid, ignore magnetic effects, or fail to account for the complex interplay between multiple nanoparticles. This study addresses these gaps by providing a **comprehensive mathematical and numerical investigation**, incorporating all relevant physical phenomena in a single framework. The study also emphasizes **thermal transport in biological systems**, which is vital for applications such as **cryotherapy, organ preservation, hyperthermia-based cancer treatments, and biosensor design**. Maintaining precise temperature ranges is critical to prevent tissue damage, edema, or burns during thermal therapies. Conventional fluids often fail to meet these requirements due to low thermal conductivity, while blood-based ternary hybrid nanofluids offer a **highly efficient and biocompatible alternative**, enhancing both heat transfer rates and safety.

In addition, **gold nanoparticles (GNPs)**, widely used in biomedical sciences, have been shown to enhance photothermal therapy, drug delivery, and contrast imaging. Their high atomic number allows for efficient heat generation under laser irradiation, selectively destroying cancer cells while minimizing damage to healthy tissues. Incorporating nanoparticles into blood enables precise control over **heat distribution, velocity profiles, and entropy generation**, which are crucial for designing safe and effective biomedical devices.

The research employs **similarity transformations** to convert the governing partial differential equations into ordinary differential equations, which are solved numerically to examine the effects of parameters such as nanoparticle volume fraction, magnetic field intensity, wall suction/injection, porosity, Grashof number, and thermal radiation. Results from the model provide insights into **velocity, temperature, and heat transfer characteristics**, which are critical for optimizing biomedical devices, including implantable sensors, thermal ablation equipment, and magnetic drug delivery systems.

Furthermore, the study explores **entropy generation and thermal efficiency**, providing a thermodynamic perspective on the flow of blood-based nanofluids through porous, magnetically influenced domains. Understanding entropy production in biological systems is essential for predicting physiological responses, optimizing therapeutic interventions, and designing energy-efficient biomedical devices.

Overall, this work provides a **comprehensive treatment of ternary hybrid nanofluids in blood**, integrating the effects of **non-Newtonian rheology, MHD, porous media, wall suction/injection, and nanoparticle interactions**. By doing so, it bridges existing gaps in literature and provides a robust framework for **enhancing heat transfer, improving therapeutic outcomes, and designing next-generation biocompatible medical devices**. The findings have broad implications for **thermal management in biomedical systems, cancer therapy, drug delivery, and bioengineering**, establishing a foundation for future research in advanced nanofluid applications.

Aims and Objectives

- To develop the governing momentum and energy equations for Maxwell hybrid nanofluid flow across a stretched surface, taking into account magnetic field effects, porous medium resistance, thermal radiation, and viscous dissipation.

- To use similarity transformations to convert partial differential equations (PDEs) into nonlinear ordinary differential equations (ODEs) that can be numerically computed.
- To numerically solve the reduced ODEs with MATLAB's `bvp4c` technique, which allows for the exploration of velocity and temperature distributions under different physical parameters.
- To investigate how critical characteristics such as the magnetic parameter (M), Eckert number (Ec), radiation parameter (Rd), Prandtl number (Pr), and permeability (Da) affect the nanofluid's velocity and temperature profiles.
- This study aims to explore the technical and biological uses of ternary hybrid nanofluids, such as BN-TiO₂-Al₂O₃ in blood, with a focus on maximizing heat transfer performance under MHD and porous circumstances

2. Literature Review

Since the revolutionary research of Choi (1995), nanofluids (fluid composed of a dispersed nanoparticle suspension in a base fluid) have proved to be quite promising in enhancing heat transfer properties and heat transfer coefficients. Initial studies were principally on mono nanoparticle suspensions, eg, AlO non-water and CuO-water. Nevertheless, there were some problems worrying about stability and thermal efficiency thereby, leading to the development of hybrid nanofluids, a blending of two types of nanoparticles. In a more recent development are ternary hybrid nanofluids (THNFs), that added three nanoparticles to them to meet a special request with a view to capitalizing on synergistic effects and provide even greater enhancements in thermophysical properties. The superior rate of heat transmission and biocompatibility of ternary hybrid nanofluids had already made them widespread in the biomedical thermal management system. Investigating the use of blood-based ternary nanofluids in biomedical settings, Ahmad et al. (2023) observed a significant improvement in heat transfer capacity and a decrease in a thickness of the boundary layer. TiO₂, Al₂O₃, and BN were often placed together because of their synergistic properties, where TiO₂ was both antibacterial and photoactive, Al₂O₃ had good chemical stability and BN had good thermal conductivity and biocompatibility. Coupled together, these nanoparticles augmented the efficacy of the nanofluid in temperature sensitive biomedical devices; such as cancer hyperthermia and targeted drug delivery. The use of blood as a base fluid was more physiologically accurate and realistic when compared to water or any other artificial fluid, in biomedical applications. It was also viscoelastic and shear thinning due to which in microcirculatory systems, it behaved as a non-Newtonian fluid. The consideration of this non-Newtonian behavior was important in coming up with accurate estimates of flow and heat transfer in biological tissues as various studies done showed including their use of the Maxwell fluid model. Other researchers, in particular Misra et al. (2017), pointed out the importance to simulate the complex blood rheology in thermal systems which took into consideration the porous and the magnetic effects. Porous media also resembled living tissues, in particular capillary networks and spongy organs. The use of modified Darcy-Brinkman models in modeling the flow of nanofluids through porous biological structures made a better description of physiological systems. Nadeem et al. (2020) and others examined, through study of hybrid nanofluid transportation within porous geometries, the important influence of

porosity on both the thermal and the velocity profile. Such modeling was useful in the design localized cooling or thermal treatment devices in biomedical engineering. According to a number of experts, all these areas thermal conductivity, rheological and mass transfer properties, suspension stability, and nanofluid heat transfer had experienced significant improvement. As a result, nanofluids played an important role in various disciplines, including biomedical engineering and healthcare.

Most of the existing literature on nanofluids aimed to analyze and understand the development of nanofluid models and their behavior in heating or cooling processes across different fields. This paper provided an in-depth discussion of nanofluids. [35] Miniature biomedical tools based on Lab-on-chip and microchannel cooling platforms are those that explored hybrid and ternary nanofluid system. In order to produce good flow through tight geometries, such systems require low viscosity fluids with high thermal conductivity. As compared to mono nanofluids and binary nanofluids, ternary nanofluids were reported to show improved heat transfer in microchannel heat sinks (Sundar et al., 2020). Their findings support the use of THNFs in small bio-thermal systems which these types of medical devices include wearable medical equipment, implanted sensors, etc. where size, performance as well as responsiveness are important criteria. Directly, nanofluids or nanoparticle (NP) emulsions/suspensions in fluids had shown great potential to enhance heat transfer in many industrial processes, such as heat exchangers, electronic and transportation systems, biomedicine, the food industry, and others. On the one hand, as the use of nanofluids in nanobiomedical technology had gained increasing popularity, their physicochemical properties had changed considerably compared with their solid counterparts. In biological applications, nanofluids had been investigated in the recent past for various purposes, such as antibacterial applications and drug delivery. [36] To the academics, mathematicians and engineers, modeling of complex phenomena to understand the mathematical physics of the same was becoming a very useful tool. Just to represent the character of intricate phenomena that emerged in mathematical physics, several crucial empirical associations had been applied and suggested.

The Maxwell model had got various important additions up to that time. As a case in point, Mushtaq conducted an analysis in terms of generalized theory of heat flow to determine the influence of buoyancy force at constant and varying wall temperatures on an elastic vertical plate. They tested the influence of a number of emergent factors on opposing and helping flow. They noted that temperature field of the system grew with energy and momentum fields of the system as a parameter of buoyancy and fluid relaxation time as an inverse parameter respectively. [37] The geometric and physical environments, such as external magnetic fields and porous media, among other things, are also influential in the dynamics of nanofluid besides the formulation of the given fluid. Biological tissues can be effectively represented as porous media, such as capillary network and spongy organs. Authors who have investigated hybrid correlations in biological tissues, e.g. Nadeem et al. (2020), highlighted the role of the porosity on the heat and momentum transport. Also, in magnetotherapy and purposeful drug delivery, the movement of nanoparticles can be controlled with magnetic fields. Other effects such as suction on the temperature and velocity distribution of the nanofluid flows are also observed and are commonly encountered in drug administration system and in surgical processes. Mathematic

models should also incorporate magnetic and suction effects as well as similarity transformation in order to correctly forecast the movement in biomedical fluids. The most important objective of the challenge was to discuss the medicinal applications of the blood-based hybrid nanofluid model through the artery past a porous channel. The nanoparticles of titanium dioxide and silver were used to prepare the hybrid nanofluid. Medical uses of this work were drug delivery, cancer, tumors in the lung, respiration, creams, wound healing and coating of medical devices with antibiotic.

According to the proposed flow model, the heat transport was stabilized by the non-Fourier theory. The system of nonlinear partial differential equations were applied to model the key phenomena. [38] Due to its multifarious K R applications in biomedical engineering, including blood dialysis in artificial kidneys, blood flow in capillaries, flow in the oxygenators, and innumerable other applications of engineering such as design of filters, porous pipe internal structure, transpiration cooling applications, and study of problem of gaseous diffusion, the flow problem in porous tubes/channels had attracted the researchers in the recent past. Boundary layer control was the process in which the decelerated fluid particles were cleared off the boundary layer and allowed to penetrate the body with the help of wall slits. Strong suction or that which provided the slowing fluid particles with additional energy would prevent separation. [39] Blood was one of the most significant multi-segment mixes observed in nature and was composed of plasma, white and red platelets, blood cells etc.

3. Methodology

3.1 Governing Equations

This section describes the governing equations, mathematical modeling, similarity transformations, numerical techniques, and parameter ranges used to investigate the flow and thermal behavior of a Maxwell ternary hybrid nanofluid in a porous biomedical domain as a function of magnetic field and surface suction.

3.2 Continuity Equation

The continuity equation is a basic formulation of mass conservation in fluid dynamics. The continuity equation for an incompressible and stable two-dimensional flow is as follows.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3.2.1)$$

3.3 Momentum Equation

Newton's second law of motion gives rise to the momentum equation, which describes the conservation of linear momentum for fluid flow. In this work, many factors impact momentum transmission inside the boundary layer: Maxwell viscoelastic behavior, magnetic field (MHD), and porous medium resistance. The momentum equation in Cartesian coordinates is:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - \lambda \left(u \frac{\partial^2 u}{\partial x \partial y} + v \frac{\partial^2 u}{\partial y^2} \right) = \nu_{\text{thnf}} \frac{\partial^2 u}{\partial y^2} - \rho_{\text{thnf}} \sigma B_0^2 u - \frac{K}{\nu_{\text{thnf}}} u \quad (3.3.1)$$

$$u = axf'(\eta), \quad v = -avf(\eta), \quad \eta = \frac{a}{v} y \quad (3.3.2)$$

$$u \frac{\partial u}{\partial x} = axf'(\eta) \cdot af'(\eta) \quad (3.3.3)$$

$$v \frac{\partial u}{\partial y} = -avf(\eta) \cdot axf''(\eta) \cdot \sqrt{\frac{a}{v}}$$

$$= - \frac{2}{\lambda} \left(\frac{1}{\sqrt{v}} \right)' \left(\frac{1}{\sqrt{v}} \right) \quad (3.3.4)$$

$$\lambda u \frac{\partial^2 u}{\partial x \partial y} = \lambda a x f'(\eta) \cdot a f''(\eta) \cdot$$

$$\sqrt{\frac{a}{v}} \quad (3.3.5)$$

$$= \lambda a^{3/2} x f'(\eta) f''(\eta) \cdot \sqrt{\frac{v}{a}} \quad (3.3.6)$$

$$\lambda v \frac{\partial^2 u}{\partial y^2} = \lambda (\sqrt{-av} f(\eta)) \cdot a x f'''(\eta) \cdot \left(\frac{v}{a} \right) \quad (3.3.6)$$

$$= -\lambda a^2 x f(\eta) f'''(\eta) \cdot \frac{\sqrt{\frac{a}{v}}}{\sqrt{v}} \quad (3.3.8)$$

$$\lambda \left(u \frac{\partial^2 u}{\partial x \partial y} + v \frac{\partial^2 u}{\partial y^2} \right) = \lambda a^2 x [f'(\eta) f''(\eta) - f(\eta) f'''(\eta)] \cdot \left(\frac{1}{\sqrt{v}} \cdot \sqrt{\frac{a}{v}} \right) \quad (3.3.9)$$

$$= \lambda a^2 x (f'(\eta) f''(\eta) - f(\eta) f'''(\eta)) \cdot \left(\frac{v}{a} \right) \quad (3.3.10)$$

$$a^2 x \left(f'^2(\eta) - f(\eta) f''(\eta) - \frac{v}{\lambda a} (f'(\eta) f''(\eta) - f(\eta) f'''(\eta)) \right) \quad (3.3.11)$$

L.H.S of momentum equation

$$v_{\text{thnf}} \frac{\partial^2 u}{\partial y^2} = v_{\text{thnf}} a x f'''(\eta) \cdot \left(\frac{v}{a} \right) \quad (3.3.12)$$

$$-\rho_{\text{thnf}} \frac{\sigma B_0^2}{K} u = -\rho_{\text{thnf}} \frac{\sigma B_0^2}{K} a x f'(\eta) \quad (3.3.13)$$

$$-\frac{K}{v_{\text{thnf}}} u = -\frac{K}{v_{\text{thnf}}} a x f'(\eta) \quad (3.3.14)$$

Maxwell Parameter:

$$\beta = \frac{\lambda a}{v}$$

(3.3.15) **Magnetic parameter:**

$$M = \frac{\sigma B_0^2}{\rho_{\text{thnf}} a} \quad (3.3.16)$$

Inverse Darcy number:

$$\frac{1}{Da} = \frac{v_{\text{thnf}}}{Ka} \quad (3.3.17)$$

$$(1 + \beta f'') f''' + f f'' - (f')^2 - \left(M + \frac{1}{Da} \right) f' = 0 \quad (3.3.18)$$

This is the final dimensionless ODE of momentum equation

3.4 Energy Equation

The energy equation is a mathematical representation of the first rule of thermodynamics for a flowing fluid, describing how thermal energy (heat) is conserved in the system. In heat transfer and fluid mechanics, especially for boundary layer flow of a nanofluid or hybrid nanofluid, the energy equation explains the temperature within the fluid.

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{\text{thnf}} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \frac{\mu}{\rho c_p} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{Q}{\rho c_p} (T - T_\infty) \quad (3.4.1)$$

$$\eta = \sqrt{\frac{a}{v}} y, \quad u = axf'(\eta), \quad v = -\sqrt{av} f(\eta), \quad \theta(\eta) = \frac{T_w - T}{T_w - T_\infty} \quad (3.4.2)$$

Rosseland Approximation for Thermal Radiation:

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y} \quad (3.4.3)$$

Linearized Form (around T_∞):

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^* T_\infty^3}{3k^*} \frac{\partial T}{\partial y} \quad (3.4.4)$$

$$u \frac{\partial T}{\partial x} = 0 \quad (3.4.5)$$

$$v \frac{\partial T}{\partial y} = -avf(\eta) \cdot (T_w - T_\infty) \theta' \cdot \frac{1}{\sqrt{v/a}} = -a(T_w - T_\infty) f(\eta) \theta' \quad (3.4.6)$$

Thermal diffusion term expressed using similarity variables.

$$\alpha_{\text{thnf}} \cdot \frac{\partial^2 T}{\partial y^2} = \alpha_{\text{thnf}} \cdot (T_w - T_\infty) \cdot \theta'' \cdot \frac{a}{v} \quad (3.4.7)$$

$$-\frac{1}{\rho c_p} \cdot \frac{\partial q_r}{\partial y} = \frac{3k^*}{16\sigma^* T_\infty^3} \cdot \rho c_p \cdot \frac{\partial^2 T}{\partial y^2} = \frac{3k^*}{16\sigma^* T_\infty^3} \cdot \rho c_p \cdot (T_w - T_\infty) \cdot \theta'' \cdot \frac{v}{a} \quad (3.4.8)$$

$$\rho c_p \mu \left(\frac{\partial u}{\partial y} \right)^2 = \rho c_p \mu \left(axf''(\eta) \cdot \sqrt{\frac{v}{a}} \right)^2 = \rho c_p \frac{v\mu}{a} x^2 (f''(\eta))^2 \quad (3.4.9)$$

$$\left(- \infty \right) = \left(- \infty \right) \left(\right)$$

Dimensionless Parameters:

Prandtl number:

$$\text{Pr} = \frac{\nu}{\alpha_{\text{thnf}}} \quad (3.4.10)$$

Radiation parameter:

$$R_d = \frac{16\sigma^* T_\infty^3}{3k^* k_{\text{thnf}}} \quad (3.4.11)$$

Eckert number:

$$\text{Ec} = \frac{a^2 x^2 \mu}{k_{\text{thnf}} (T_w - T_\infty)} \quad (3.4.12)$$

Heat source/sink parameter:

$$Q^* = \frac{Q}{\rho c_p a} \quad (3.4.13)$$

$$\frac{\nu}{\alpha_{\text{thnf}}} = \text{Pr} \quad \Rightarrow \quad \frac{\alpha_{\text{thnf}}}{\nu} = \frac{1}{\text{Pr}} \quad (3.4.14)$$

Dimensionless energy equation with thermal radiation, viscous dissipation, and heat source/sink:

$$(1 +)' + \text{Pr} (+ \text{Ec} ()^2 + *) = 0 \quad (3.4.15)$$

Boundary Conditions:

$$(0) = 1, \quad (\infty) \rightarrow 0 \quad (3.4.16)$$

The final dimensionless energy equation takes into account thermal radiation, viscous dissipation, and internal heat generation/absorption. Dimensionless Energy Equation:

$$(1 +)' + \text{Pr} (+ \text{Ec} ()^2 + *) = 0 \quad (3.4.17)$$

Now **Dimensionless Momentum Equation (Maxwell + MHD + Porous**

Medium):

$$(1 + \beta f'')f''' + ff'' - (f')^2 - \left(M + \frac{1}{Da}\right)f' = 0 \quad (3.4.18)$$

Where:

- $(\cdot)$ is the dimensionless stream function,
- β is the Maxwell fluid parameter,
- M is the magnetic parameter (Hartmann number),
- Da is the Darcy number (inversely related to porous resistance).

Boundary Conditions for Flow over a Stretching Surface:

$$\begin{aligned} f(0) = 0 \quad (\text{noslip at surface}), \quad f'(0) = 1 \quad (\text{stretching velocity}), \quad (3.4.19) \\ f'(\infty) \rightarrow 0 \quad (\text{velocity vanishes far from surface}) \end{aligned}$$

Transformation to First-Order System for Shooting Method:

Let:

$$\begin{aligned} y_1 &= f, \quad y_2 = f', \quad y_3 = f'' \\ y_1' &= y_2 \\ y_2' &= y_3 \\ y_3' &= \frac{1}{1+\beta y_3} \left[-y_1 y_3 + y_2^2 + \left(M + \frac{1}{Da}\right) y_2 \right] \end{aligned} \quad (3.4.20)$$

Boundary Conditions:

$$y_1(0) = 0, \quad y_2(0) = 1, \quad y_3(\infty) \rightarrow 0$$

First-Order System for the Shooting Method:

$$\begin{aligned} \frac{dy_1}{d\eta} &= y_2 \\ \frac{dy_2}{d\eta} &= y_3 \end{aligned}$$

$$\frac{dy_3}{d\eta} = \frac{1}{1+\beta y_3} \left(-y_1 y_3 + y_2^2 + \left(M + \frac{1}{Da}\right) y_2 \right) \quad (3.4.21)$$

Shooting Method Setup

We reduce the third-order ODE to a system:

$$\begin{aligned} \frac{dy_1}{d\eta} &= y_2, \quad \frac{dy_2}{d\eta} = y_3, \quad \frac{dy_3}{d\eta} = \frac{1}{1+\beta y_3} \left(-y_1 y_3 + y_2^2 + \left(M + \frac{1}{Da}\right) y_2 \right) \end{aligned} \quad (3.4.22)$$

Initial Conditions (for IVP):

$$y_1(0) = 0, \quad y_2(0) = 1, \quad y_3(0) = \text{guess} \quad (3.4.23)$$

Shooting Goal:

Guess $y_3(0)$ and integrate up to $\eta = 6$. Adjust $y_3(0)$ until the boundary condition is satisfied:

$$y_2(6) \approx y_2(\infty) \rightarrow 0$$

Now we use similarity transformation on momentum equation and energy equation then we have both equations

3.4.1 Momentum Equation

$$(1 + \beta f'')f''' + ff'' - (f')^2 - \left(M + \frac{1}{Da}\right)f' = 0 \quad (3.4.24)$$

3.4.2 Energy Equation

$$(1 + \beta f'')f''' + ff'' - (f')^2 - \left(M + \frac{1}{Da}\right)f' = 0 \quad (3.4.25)$$

4. Results and Discussion

This section presents the numerical results for the Cu–Al₂O₃/water hybrid nanofluid obtained using MATLAB's bvp4c solver. The effects of key physical parameters—including magnetic parameter (M), thermal radiation parameter (Rd), Eckert number (Ec), heat generation, suction parameter, and channel expansion ratio—on velocity, temperature, entropy generation, and the Bejan number are analyzed.

4.1 Velocity Profile

The velocity distribution is strongly influenced by the magnetic field and suction at the wall. Increasing the magnetic parameter reduces the velocity due to the Lorentz force, which opposes fluid motion and creates electromagnetic drag, consistent with magnetohydrodynamic (MHD) theory. Higher suction leads to a thinner boundary layer and lower velocity by removing fluid near the wall, enhancing flow stability. Additionally, channel contraction accelerates the fluid, while expansion slows it down.

4.2 Temperature Distribution

The temperature profile rises with increasing Eckert number, reflecting enhanced viscous dissipation, which converts kinetic energy into internal energy. Higher thermal radiation (Rd) and heat generation parameters further increase fluid temperature by improving heat transfer and introducing internal thermal energy, respectively, leading to a thicker thermal boundary layer.

4.3 Entropy Generation

Entropy generation increases with both the magnetic parameter and Eckert number. A stronger magnetic field enhances Ohmic and viscous dissipation, increasing irreversibility, while higher viscous effects at larger Ec also contribute to increased entropy.

4.4 Bejan Number

The Bejan number, representing the ratio of heat transfer irreversibility to total entropy generation, decreases with increasing M and Ec, indicating that viscous dissipation dominates over thermal irreversibility in the system.

Overall, the study shows that magnetic fields, wall suction, viscous dissipation, thermal radiation, and internal heat generation significantly influence the flow and thermal behavior of the hybrid nanofluid, which is critical for optimizing heat transfer and fluid transport in biomedical and engineering applications.

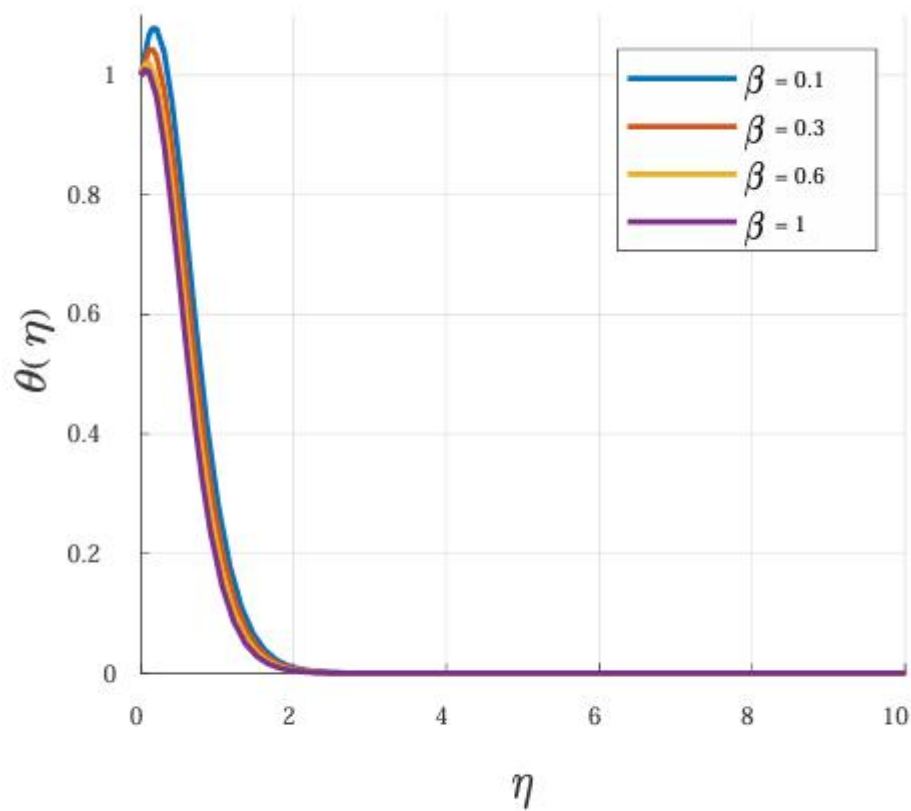
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Fig(4.1): Variation of Temperature Profile with Maxwell Parameter

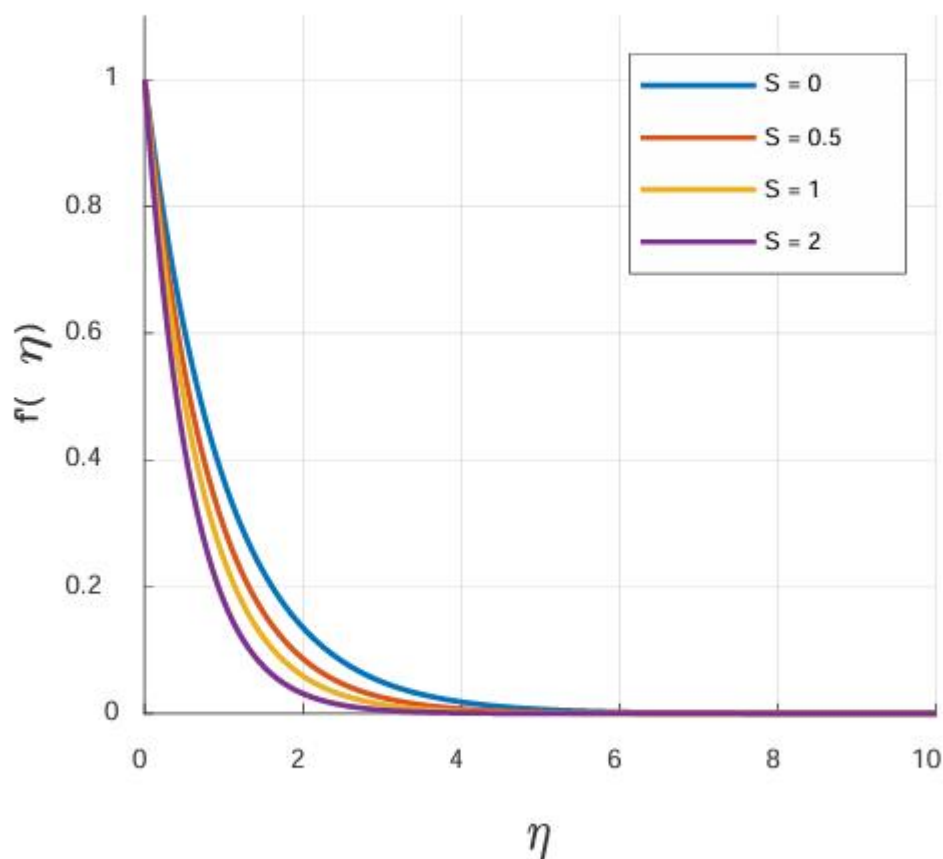
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Fig(4.2): Velocity Profile for Different Suction Parameter S

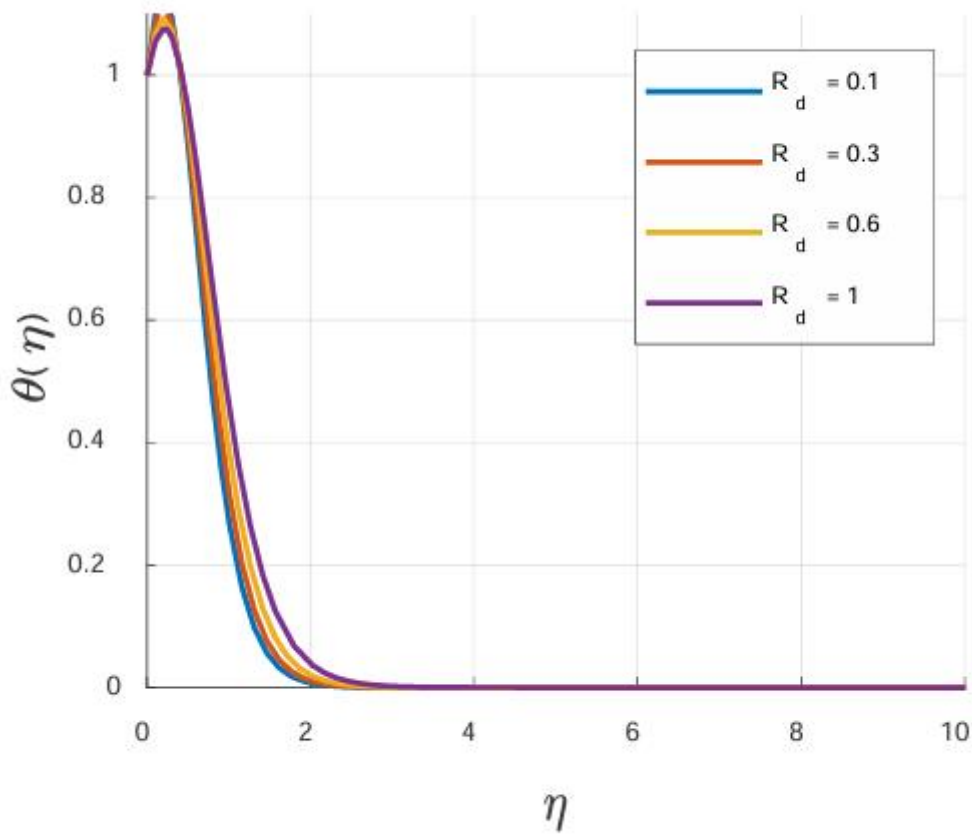
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Fig(4.3): Temprature profile with Rd

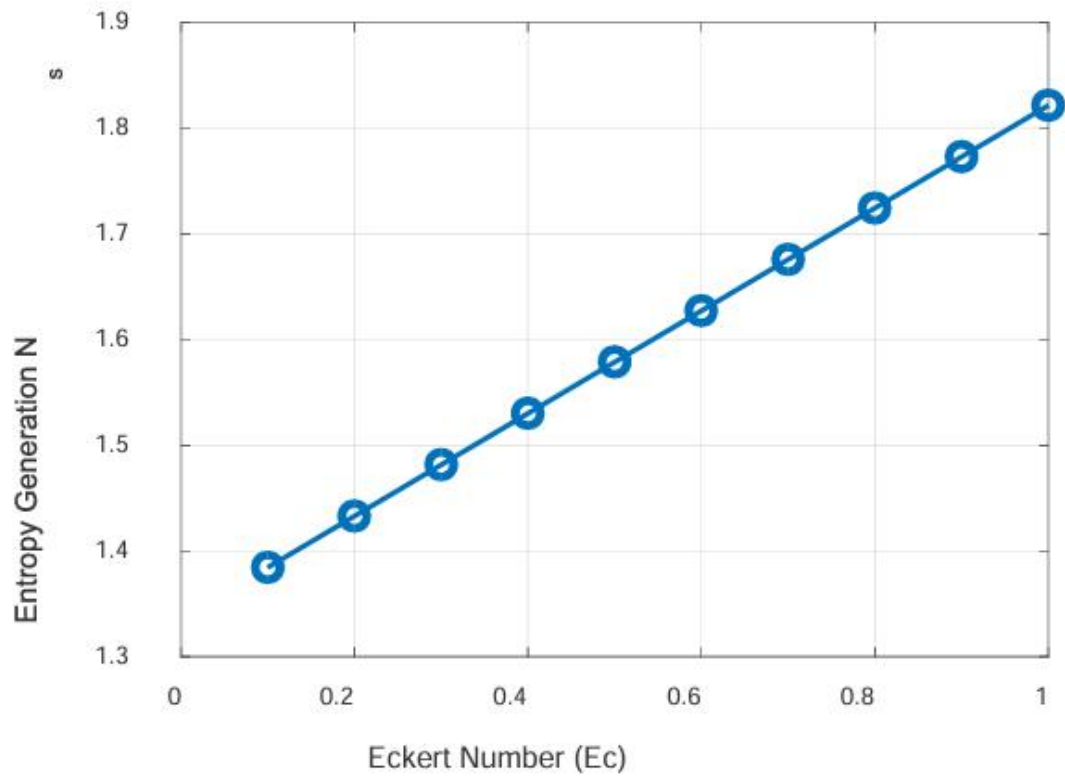
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Fig(4.4): Variation of Entropy Generation with Magnetic Field

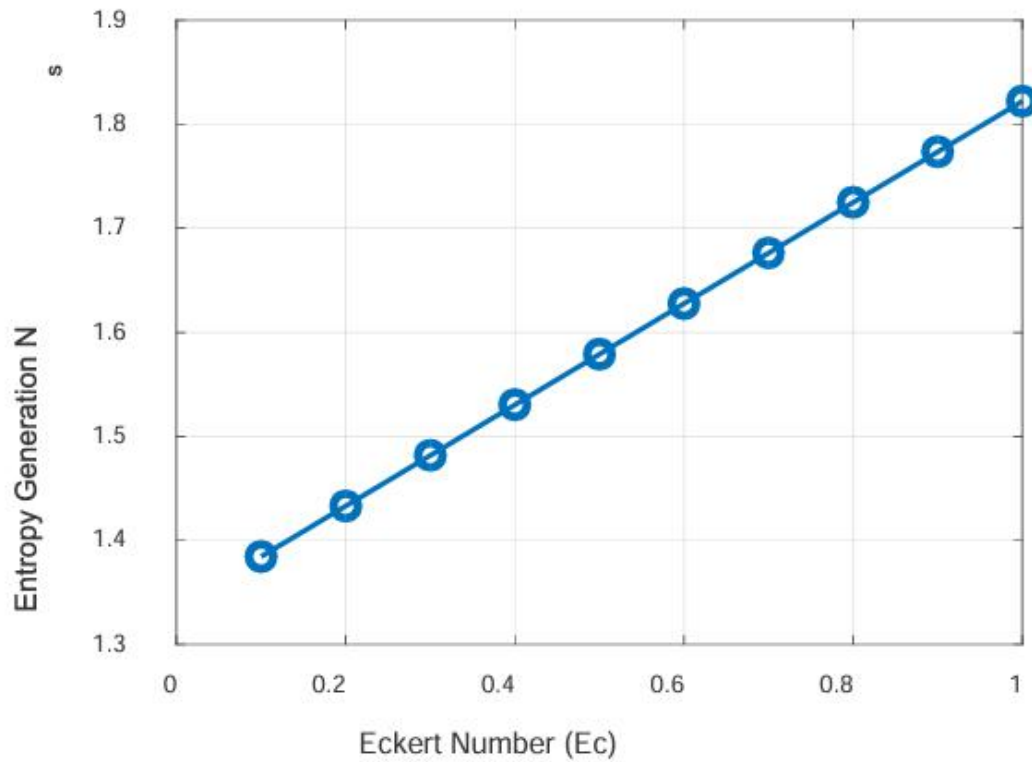
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Fig(4.5): Variation of Entropy Generation with Eckert Number

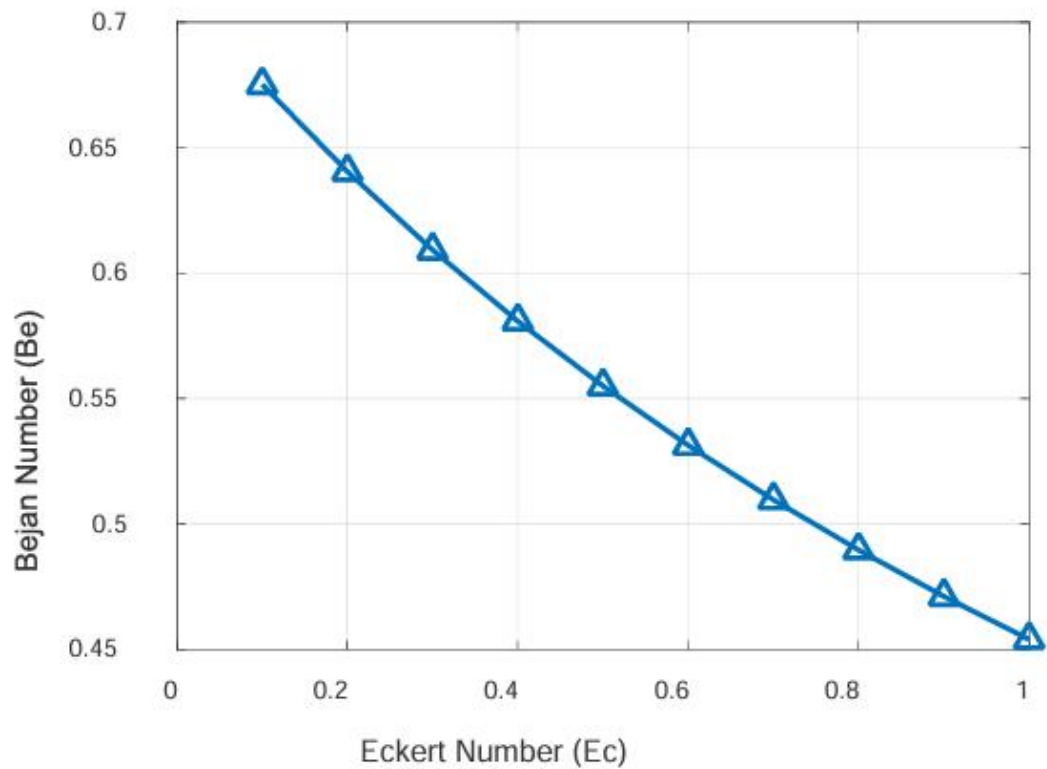
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Fig(4.6): Bejan Number vs. Eckert Number

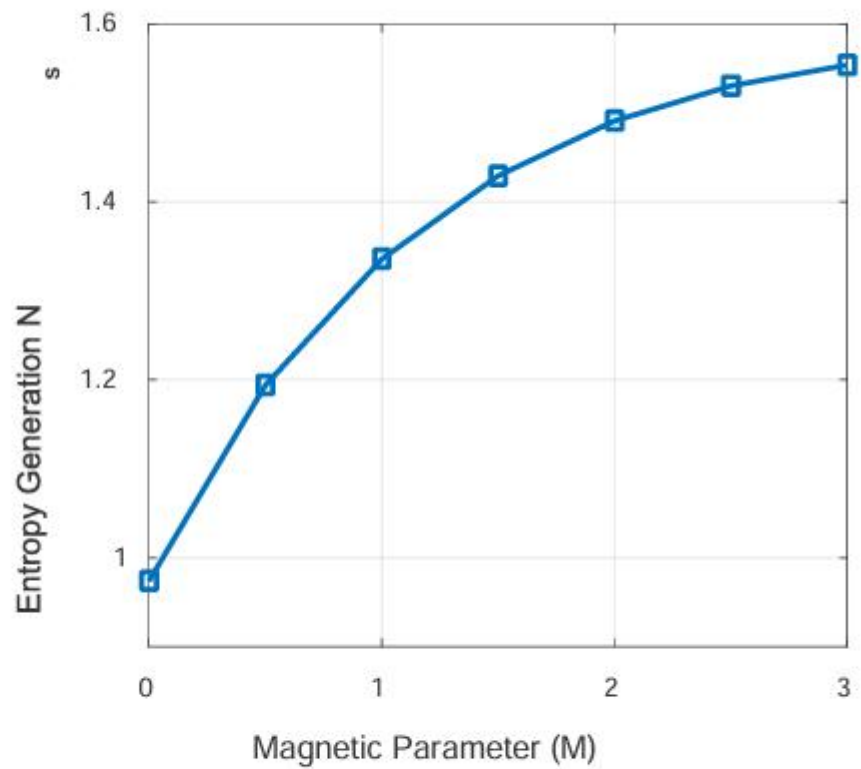
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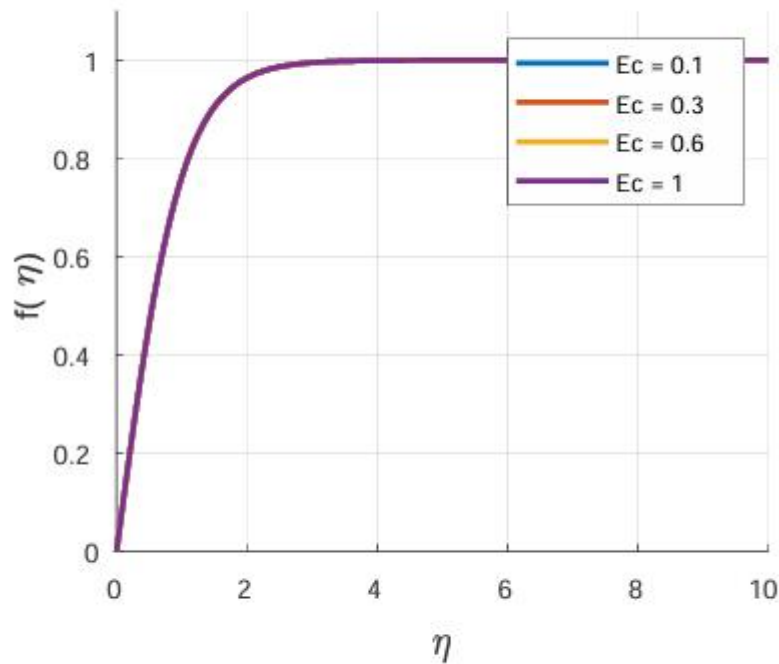
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Fig(4.7): Variation of Entropy Generation with Magnetic Field



Fig(4.8):Stream function varying EC

5.1 Discussion

The study demonstrates that channel geometry, viscous dissipation, thermal radiation, and magnetic effects significantly influence the flow and thermal behavior of hybrid nanofluids. A contracting channel accelerates fluid motion, increasing velocity, whereas an expanding channel slows it down due to divergent flow. Higher Eckert numbers elevate fluid temperature by converting mechanical energy into thermal energy through viscous dissipation, thickening the thermal boundary layer—an effect relevant in energy transport and hyperthermia treatments. Increased thermal radiation also raises fluid temperature by enhancing radiative heat transfer, which is particularly important in high-temperature or radiative environments such as solar collectors or blood flow in radiated tissues.

Entropy generation rises with both the magnetic parameter and Eckert number, reflecting increased irreversibility due to Ohmic and viscous dissipation. Correspondingly, the Bejan number decreases, indicating that fluid friction dominates over thermal irreversibility. These findings highlight the interplay between electromagnetic, thermal, and viscous effects in optimizing flow and energy transport in hybrid nanofluids.

5.2 Conclusions

This work analyzed the flow and thermal characteristics of a Maxwell ternary

hybrid nanofluid (BN–TiO₂–Al₂O₃) in a porous biomedical domain under the influence of magnetic fields, suction, and thermal radiation using MATLAB's bvp4c solver. Key conclusions include:

- **Magnetic effects:** Lorentz forces reduce fluid velocity, enhancing thermal control but increasing entropy generation.
- **Nanoparticle effects:** Ternary nanoparticles improve thermal conductivity, boosting energy transfer in biomedical applications.
- **Wall suction:** Promotes thinner boundary layers, stable flow, and better thermal management.
- **Thermal radiation and viscous dissipation:** Significantly impact temperature profiles, crucial for hyperthermia and energy transport.
- **Entropy analysis:** Increased magnetic and viscous effects raise system irreversibility, reducing thermal efficiency.

Overall, Maxwell ternary hybrid nanofluids show strong potential for biomedical applications, including targeted drug delivery, hyperthermia treatment, and temperature regulation in implants. The combination of non-Newtonian fluid behavior, porous tissue modeling, and MHD effects provides a realistic foundation for designing advanced biomedical devices and optimizing thermal systems.

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