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Travelling Wave Solutions Of Nonlinear Time-Fractional Partial Differential Equations Via $\left(\frac{G'}{G}\right)$- Expansion Method

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ABSTRACT

Now a day's fractional calculus concerns with differentiations and integrations of non-integer order has received great attentions of researchers. Fractional differential equations both ordinary and partial play very important role in mathematical modeling in different areas of engineering and science like acoustics, electromagnetism, mathematical biology, fluid mechanics, solid state physics, signal processing quantum theory, diffusion and many other physical phenomenon. It is essential to obtain the closed-form solutions of these fractional differential equations (FDE's) to be aware of the complete physical phenomenon of the governing mathematical model. In literature many mathematical methods has been represented for solving linear FDE's, however it is still of great interest for researchers to achieve the closed-form solutions of non-linear time fractional partial differential equations (NTFPDE's). In this, we shall derive the closed-form solutions of NTFPDE's by $\left(\frac{G'}{G}\right)$-expansion method. This method will reduce the NTFPDE's to non-linear ordinary differential equation (ODE), and then by using expansion series the non-linear ODE will be converted to nonlinear algebraic system of equations. By solving such system for required parameters, finally we will get closed-form solution of nonlinear FDE as to Trigonometric, Hyperbolic and Rational functions. Finally the graphical illustration of obtained solutions will be presented by using Maple.

Keywords and phrases: Space-Time Fractional Differential Equations, $\left(\frac{G'}{G}\right)$ - Expansion Method, Non-Linear FDE, Exact Solutions.

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INTRODUCTION

Investigating exact solutions of non-linear evolution equations holds significant importance in understanding non-linear physical phenomenon. In the latest years, physicists and mathematicians alike have invested significant efforts into exploring closed-form solutions for non-linear ordinary DE's or partial DE's with fractional order. They have been looking into how these solutions relate to the real- life phenomenon in science and engineering areas. In a literature, various effective methods been have presented. For instance modified simple method [1, 2], Adomian decomposition method [3, 4], Modified extended Tanh method [5, 6], Tanh-Coth method [8, 9], quadratic Jacobi's elliptic function expansion [10, 11], Modified **(**G'/G)-Expansion method [12], Homotopy perturbation method [13, 14], Residual power series method [15, 16], Homotopy perturbation method [17,18], Variational iteration method and some modification [19, 21], The finite element technique [22, 23], The finite difference method [24, 25], The exp-function method and various modification [26, 27], the GERF method [28, 29], Bäcklund transformation method [30], Hetero-Bäcklund transformations [31], Simple equations method (SEM) and its connection with the inverse scattering transform method [32], Inverse scattering method [33], the generalized Kudryashov method [34, 35]. The goal of this article is to apply a method known as the $\left(\frac{G'}{G}\right)$-expansion method, along with the computer algebraic system Maple, to create several exact solitons solutions for specific kinds of following non-linear NTFPDE's.

The first model considered is the time fractional Gardner equation (TFGE) arises in scientific applications wave phenomena like plasma physics, nonlinear optics, fluid dynamics, and so on [36]. The Gardner equation is given by

$$(1) \quad D_t^{\alpha} u + 2\delta u u_x + 3\beta u^2 u_x + \gamma u_{xxx} = 0; \quad 0 < \alpha \leq 1, t > 0$$

where $u(x,t)$ is the wave amplitude, representing the profile of the wave, t is time, x is spatial position δ, β and γ are arbitrary parameters.

The second model studied is the time fractional Benjamin-Bona Mahoney (TFBBM) equation has application in different fields of engineering and physics, including plasma physics, fluid dynamics and it provides a more accurate description of wave propagation in certain physical systems where memory effects or long-range interaction play a significant role [37]. The (BBM) model is given by

$$(2) \quad D_t^{\alpha} u + u_x + u u_x - u_{xx}(D_t^{\alpha} u) = 0; \quad 0 < \alpha \leq 1, t > 0$$

Finally, the third equation that we need to solve is time fractional Cahn-Allen (TFCA) equation is used in many fields of science and engineering like quantum mechanics, plasma physics, solid state physics and mathematical biology, [38, 39]. The Cahn-Allen model is given by

$$(3) \quad D_t^{\alpha} u - u_{xx} + u^3 - u = 0; \quad 0 < \alpha \leq 1, t > 0$$

The rest of this work is set up like this: in section 2, Representation of fundamental properties of the modified Riemann-Liouville derivatives operator. In section 3 the approach steps of the $\left(\frac{G'}{G}\right)$-expansion. In section 4 to 5, we implement $\left(\frac{G'}{G}\right)$-expansion approach in detail together time fractional TFGE, BBME and TFCAE. In last section

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graphs of solutions and some results are provided.

THE MODIFIED RIEMANN-LIOUVILLE DERIVATIVE AND PROPERTIES

This letter describes method that $\left(\frac{G'}{G}\right)$-expansion method to achieve exact solitons solutions involving time fractional order derivatives in non-linear differential equations with modified Riemann-Liouville derivative order α is defined as [40]

$$(4)\quad D_t^{\alpha} h(t) = \begin{cases} \frac{1}{\Gamma(-\alpha+1)} \int_0^t (t-\xi)^{-1-\alpha}[h(\xi)-h(0)]d\xi, & \text{if } \alpha < 0, \\ \frac{1}{\Gamma(-\alpha+1)} \frac{d}{dt} \int_0^t (t-\xi)^{-\alpha}[h(\xi)-h(0)]d\xi, & \text{if } 0 < \alpha < 1, \\ [h^{(\alpha-n)}(t)]^{(n)}, & \text{if } n \le \alpha < n+1,\ n \ge 1. \end{cases}$$

In which Γ (*) is the Gamma function defined as

$$(5)\quad \Gamma(\alpha) = \lim_{n\to\infty} \frac{n!\, n^{\alpha}}{(\alpha+1)(\alpha+2)\ldots(\alpha+n)}$$

Properties

$$(6)\quad D_t^{\alpha} t^r = \frac{\Gamma(1+r)}{\Gamma(r-\alpha+1)} t^{r-\alpha}$$

$$(7)\quad D_t^{\alpha}[h(t)g(t)] = g(t)D_t^{\alpha}h(t) + h(t)D_t^{\alpha}g(t)$$

$$(8)\quad D_t^{\alpha}h[g(t)] = \frac{dh(g(t))}{dg(t)} D_t^{\alpha}g(t)$$

DESCRIPTION OF THE $\left(\frac{G'}{G}\right)$-EXPANSION METHOD

In way to understand the $\left(\frac{G'}{G}\right)$-expansion method, suppose the following non-linear TFDE in two independent variables x and t in the form

$$(9)\quad R(u, u_x, u_t, u_{xx}, \ldots, D_t^{\alpha}u, \ldots) = 0, \qquad 0 < a \le 1$$

Where $D_x^{\alpha}u$ are fractional derivatives operator of u, $u(x,t)$ is unknown function, R is a polynomial in u, and its various fractional derivatives and non-linear terms are concerned with linear terms

Step 1: Let us consider travelling wave transformation as follows

$$(10)\quad u(x,t) = u(\xi), \text{ where } \xi = kx - \frac{ct^{\alpha}}{\Gamma(1+\alpha)}$$

Where $k, c \neq 0$ constants to be determined later. The soliton wave variable Eq. (9) allows to reduce Eq. (8) into non-linear ODE for $G = G(\xi)$ as,

$$(11)\quad R(u, ku', -cu', k^2u'', \ldots, -cu', \ldots) = 0,$$

Step 2: Integrating Eq. (11) if it contains derivatives in each term and assume constant of integration is zero.

Step 3: Assume the solution of non-linear ODE given in Eq. (11) can be written through a polynomial in $\left(\frac{G'}{G}\right)$ as follow

$$(12)\quad u(\xi) = \sum_{i=0}^{m} a_i \left(\frac{G'}{G}\right)^i, \quad a_m \neq 0,$$

where $a_i (i = 0,1,2 \ldots, m)$ are constants to determine latter although $G = G(\xi)$ will satisfy the following second order linear ODE

$$(13)\quad G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0$$

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Where $G' = \frac{dG}{d\xi}$, $G'' = \frac{dG^2}{d\xi^2}$, λ and μ are random parameters to determine later. Now, positive integer m may be found by assuming the homogeneity balance between non-linear terms and highest order derivative occurring in Eq. (11). To the letter, we describe the degree of $u(\xi)$ like $\mathrm{Deg}[u(\xi)] = m$ that leads to the degree of the other expression such as [41]

(14) $\deg\left[\frac{d^p U}{d\xi^p}\right] = m + p$

(15) $\deg\left[U^s \left(\frac{d^p U}{d\xi^p}\right)^q\right] ms + q(p + m)$

By the generalised solution of Eq. (13), we get [42]

(16) $$\left(\frac{G'}{G}\right) = \begin{cases} -\frac{\lambda}{2} + \frac{\sqrt{\lambda^2 - 4\mu}}{2}\left(\frac{c_1 \sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right) + c_2 \cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}{c_1 \cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right) + c_2 \sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}\right), & \lambda^2 - 4\mu > 0 \\ -\frac{\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-c_1 \sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right) + c_2 \cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)}{c_1 \cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right) + c_2 \sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)}\right), & \lambda^2 - 4\mu < 0 \\ -\frac{\lambda}{2} + \frac{c_2}{c_1 + c_2 \xi}, & \lambda^2 - 4\mu = 0 \end{cases}$$

Where c_1 and c_2 are orbitrary constants.

Step 4: Plugging Eq. (12) with Eq. (13) into the Eq. (11), combining all terms of identical power $\left(\frac{G'}{G}\right)$, then Eq. (11) is changed within another polynomial in $\left(\frac{G'}{G}\right)$, equate each coefficient of this polynomial to zero and a system of algebraic equations for $a_i (i = 0,1,2 \ldots, m)$
λ, k, c and μ will be obtained.

Step 5: From algebraic system in step 4, we can find constants $a_i (i = 0,1,2 \ldots, m)$, $, \lambda, k, c$ and μ with the help of maple. Since the generalized solution of LODE (13) have been well-known for us, then plugging $a_i (i = 0,1,2 \ldots, m), k$ and c into the generalized solutions of Eq. (13) into Eq. (12), we get a series of fundamental solutions of the nonlinear evaluation Eq. (9).

APPLICATIONS

By the implementation of $\left(\frac{G'}{G}\right)$-expansion approach, we obtained the exact solitons solutions of three well-known equations.

THE TIME FRACTIONAL GARDNER EQUATION

We consider soliton wave variable of Eq. (1) and perform the transformation Eq. (10), and the fractional parameter α lies $0 < \alpha \leq 1$. By plugging Eq. (10) in Eq. (1), the equation can be reduce to non-linear ODE.

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(17) $-cu' + 2\delta k u u' + 3\beta k\, u^2 u' + \gamma k^3 u''' = 0$

Integrate Eq. (17) with respect to ξ and suppose integration constant is equal to zero

(18) $-cu + \delta k\, u^2 + \beta k u^3 + \gamma k^3 u'' = 0$

Suppose Eq. (18) has the solution is Eq. (12). By using Eq. (14) and Eq. (15) the homogenous balance between u^3 and u'' is

$$2 + m = 3m \Rightarrow m = 1$$

Thus from Eq. (12), we have

(19) $u(\xi) = a_0 + a_1\left(\frac{G'}{G}\right), \qquad a_1 \neq 0$

(20) $u^2 = {a_0}^2 + 2a_0a_1\left(\frac{G'}{G}\right) + {a_1}^2\left(\frac{G'}{G}\right)^2$

(21) $u^3 = {a_0}^3 + 3{a_0}^2a_1\left(\frac{G'}{G}\right) + 3a_0{a_1}^2\left(\frac{G'}{G}\right)^2 + {a_1}^3\left(\frac{G'}{G}\right)^3$

Through Eq. (19) and Eq. (13), we get

(22) $u'' = 2a_1\left(\frac{G'}{G}\right)^3 + 3a_1\lambda\left(\frac{G'}{G}\right)^2 + (2a_1\mu + a_1\lambda^2)\left(\frac{G'}{G}\right) + a_1\lambda\mu$

By plugging Eq. (19)-Eq. (22) into Eq. (18) and by equate coefficients of identical power of $\left(\frac{G'}{G}\right)$, we get the system of algebraic equations for a_0, a_1, k and c as follows:

$$\left(\frac{G'}{G}\right)^0 : -ca_0 + \delta k {a_0}^2 + {a_0}^3\beta k + a_1\lambda\mu\gamma k^3 = 0$$

$$\left(\frac{G'}{G}\right)^1 : -ca_1 + 2a_0a_1\delta k + 3{a_0}^2a_1\beta k + 2a_1\mu\gamma k^3 + a_1\lambda^2\gamma k^3 = 0$$

$$\left(\frac{G'}{G}\right)^2 : {a_1}^2\delta k + 3a_0{a_1}^2\beta k + 3a_1\lambda\gamma k^3 = 0$$

$$\left(\frac{G'}{G}\right)^3 : {a_1}^3\beta k + 2a_1\gamma k^3 = 0$$

After solving the above algebraic equations, the constants, a_0, a_1, c and k can be achieved with the help of maple 13, we have one set of solution

(23) $a_0 = -\frac{\left(\lambda+\sqrt{\lambda^2-4\mu}\right)\delta}{3\beta\sqrt{\lambda^2-4\mu}}, a_1 = -\frac{2\delta}{3\beta\sqrt{\lambda^2-4\mu}}, k = \frac{\sqrt{2}\delta}{3\sqrt{\gamma\beta}.\sqrt{4\mu-\lambda^2}}, c = -\frac{2\sqrt{2}\delta^3}{27\sqrt{\gamma}\ \beta^{\frac{3}{2}}\sqrt{4\mu-\lambda^2}}$

Where λ, μ, δ, γ and β are constant parameters, by using Eq. (23), the equation (19) can be expressed as

(24) $u(\xi) = -\frac{(\lambda+\sqrt{\lambda^2-4\mu})\delta}{3\beta\sqrt{\lambda^2-4\mu}} - \frac{2\delta}{3\beta\sqrt{\lambda^2-4\mu}}\left(\frac{G'}{G}\right)$

Where $\xi = \frac{\sqrt{2}\delta x}{3\sqrt{\gamma\beta}.\sqrt{4\mu-\lambda^2}} + \left(\frac{2\sqrt{2}\delta^3}{27\sqrt{\gamma}\ \beta^{\frac{3}{2}}\sqrt{4\mu-\lambda^2}}\right)\frac{t^\alpha}{\Gamma(1+\alpha)}$.

Plugging the generalize solutions Eq. (16) into Eq. (24), we get different kinds of travelling wave solutions of the Eq. (1)

Case I: If $\lambda^2 - 4\mu > 0$

$u(\xi) =$

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$$-\frac{(\lambda+\sqrt{\lambda^2-4\mu})\delta}{3\beta\sqrt{\lambda^2-4\mu}}-\frac{2\delta}{3\beta\sqrt{\lambda^2-4\mu}}\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{c_1\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)+c_2\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}{c_1\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)+c_2\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}\right)\right)$$

Case II: If $\lambda^2-4\mu<0$

$u(\xi)=$

$$-\frac{(\lambda+\sqrt{\lambda^2-4\mu})\delta}{3\beta\sqrt{\lambda^2-4\mu}}-\frac{2\delta}{3\beta\sqrt{\lambda^2-4\mu}}\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-c_1\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)+c_2\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)}{c_1\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)+c_2\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)}\right)\right)$$

Case III: If $\lambda^2-4\mu=0$

$$u(\xi)=-\frac{(\lambda+\sqrt{\lambda^2-4\mu})\delta}{3\beta\sqrt{\lambda^2-4\mu}}-\frac{2\delta}{3\beta\sqrt{\lambda^2-4\mu}}\left(-\frac{\lambda}{2}+\frac{c_2}{c_1+c_2\xi}\right)$$

Where $\xi=\frac{\sqrt{2}\delta x}{3\sqrt{\gamma\beta}.\sqrt{4\mu-\lambda^2}}+\left(\frac{2\sqrt{2}\delta^3}{27\sqrt{\gamma}\,\beta^{\frac{3}{2}}\sqrt{4\mu-\lambda^2}}\right)\frac{t^\alpha}{\Gamma(1+\alpha)}$ for all above cases and c_1 and c_2 are arbitrary parameters.

Case I: If put $c_1=0$, then

$$u_1=-\frac{\delta}{3\beta}\left(1+\coth\left(\frac{\sqrt{2}\delta}{6\sqrt{\gamma\beta}}x+\frac{\sqrt{2}\delta^3\,t^\alpha}{27\sqrt{\gamma}\,\beta^{\frac{3}{2}}\Gamma(1+\alpha)}\right)\right)$$

And If $c_2=0$, then

$$u_2=-\frac{\delta}{3\beta}\left(1+\tanh\left(\frac{\sqrt{2}\delta}{6\sqrt{\gamma\beta}}x+\frac{\sqrt{2}\delta^3\,t^\alpha}{27\sqrt{\gamma}\,\beta^{\frac{3}{2}}\Gamma(1+\alpha)}\right)\right)$$

Case II: If put $c_1=0$, then

$$u_3=-\frac{\delta}{3\beta}\left(1+\cot\left(\frac{\sqrt{2}\delta}{6\sqrt{\gamma\beta}}x+\frac{\sqrt{2}\delta^3\,t^\alpha}{27\sqrt{\gamma}\,\beta^{\frac{3}{2}}\Gamma(1+\alpha)}\right)\right)$$

And If $c_2=0$, then

$$u_4=-\frac{\delta}{3\beta}\left(1-\tan\left(\frac{\sqrt{2}\delta}{6\sqrt{\gamma\beta}}x+\frac{\sqrt{2}\delta^3\,t^\alpha}{27\sqrt{\gamma}\,\beta^{\frac{3}{2}}\Gamma(1+\alpha)}\right)\right)$$

THE TIME FRACTIONAL BENJAMIN-BONA MAHONEY (BBM) EQUATION

We consider soliton wave variable of Eq. (2) and perform the transformation Eq. (10). By plugging Eq. (10) in Eq. (2), the equation can be reduce to non-linear ODE.

(25) $\quad -cu'+ku'+\frac{k}{2}(u^2)'+ck^2u'''=0$

Integrate Eq. (25) with respect to ξ and suppose integration constant is equal to zero

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(26) $$-cu + \mathrm{k}u + \frac{\mathrm{k}}{2}u^2 + \mathrm{ck}^2u'' = 0$$

Suppose Eq. (26) has the solution Eq. (12). By using Eq. (14) and Eq. (15) the homogenous balance between u^2 and u'' is

$$2 + m = 2\mathrm{m} \Rightarrow \mathrm{m} = 2$$

Thus from Eq. (12), we have

(27) $$u(\xi) = a_0 + a_1\left(\frac{\mathrm{G}'}{\mathrm{G}}\right) + a_2\left(\frac{\mathrm{G}'}{\mathrm{G}}\right)^2 \qquad a_2 \neq 0$$

(28) $$u^2 = {a_0}^2 + {a_1}^2\left(\frac{\mathrm{G}'}{\mathrm{G}}\right)^2 + {a_2}^2\left(\frac{\mathrm{G}'}{\mathrm{G}}\right)^4 + 2a_0a_1\left(\frac{\mathrm{G}'}{\mathrm{G}}\right) + 2a_1a_2\left(\frac{\mathrm{G}'}{\mathrm{G}}\right)^3 + 2a_0a_2\left(\frac{\mathrm{G}'}{\mathrm{G}}\right)^2$$

By using Eq. (27) and Eq. (13), we get

(29) $$u'' = 6a_2\left(\frac{\mathrm{G}'}{\mathrm{G}}\right)^4 + (10a_2\lambda + 2a_1)\left(\frac{\mathrm{G}'}{\mathrm{G}}\right)^3 + (3a_1\lambda + 8a_2\mu + 4a_2\lambda^2)\left(\frac{\mathrm{G}'}{\mathrm{G}}\right)^2 + (2a_1\mu + a_1\lambda^2 + 6a_2\lambda\mu) + \left(\frac{\mathrm{G}'}{\mathrm{G}}\right) + 2a_2\mu^2 + a_1\lambda\mu$$

By plugging Eq. (27)-Eq. (29) into Eq. (26) and by equate coefficients of identical power of $\left(\frac{\mathrm{G}'}{\mathrm{G}}\right)$, we get the system of algebraic equations for a_0, a_1, a_2, k and c as follows:

$$\left(\frac{\mathrm{G}'}{\mathrm{G}}\right)^0 : -ca_0 + \mathrm{k}a_0 + \frac{{a_0}^2}{2}\mathrm{k} + a_1\lambda\mu c\mathrm{k}^2 + 2a_2\mu^2c\mathrm{k}^2 = 0$$

$$\left(\frac{\mathrm{G}'}{\mathrm{G}}\right)^1 : -ca_1 + \mathrm{k}a_1 + \mathrm{k}a_0a_1 + 6a_2\lambda\mu c\mathrm{k}^2 + 2a_1\mu c\mathrm{k}^2 + a_1\lambda^2c\mathrm{k}^2 = 0$$

$$\left(\frac{\mathrm{G}'}{\mathrm{G}}\right)^2 : -ca_2 + \mathrm{k}a_2 + \frac{{a_1}^2}{2}\mathrm{k} + \mathrm{k}a_0a_2 + 8a_2\mu c\mathrm{k}^2 + 3a_1\lambda c\mathrm{k}^2 + 4a_2\lambda^2c\mathrm{k}^2 = 0$$

$$\left(\frac{\mathrm{G}'}{\mathrm{G}}\right)^3 : \mathrm{k}a_1a_2 + 2a_1\mathrm{ck}^2 + 10a_2\lambda c\mathrm{k}^2 = 0$$

$$\left(\frac{\mathrm{G}'}{\mathrm{G}}\right)^4 : \frac{{a_2}^2}{2}\mathrm{k} + 6a_2c\mathrm{k}^2 = 0$$

After solving the above algebraic equations, the constants, a_0, a_1, a_2, c and k can be achieved with the help of maple 13, we have one set of solution

(30) $$\mathrm{k} = 1, c = \frac{1}{4\mu + 1 - \lambda^2}, a_0 = -\frac{12\mu}{4\mu + 1 - \lambda^2}, a_1 = -\frac{12\lambda}{4\mu + 1 - \lambda^2}, a_2 = \frac{12}{4\mu + 1 - \lambda^2}$$

Where λ and μ are constant parameters, by using Eq. (30), the equation (27) can be expressed as

(31) $$u(\xi) = -\frac{12\mu}{4\mu + 1 - \lambda^2} - \frac{12\lambda}{4\mu + 1 - \lambda^2}\left(\frac{\mathrm{G}'}{\mathrm{G}}\right) + \frac{12}{4\mu + 1 - \lambda^2}\left(\frac{\mathrm{G}'}{\mathrm{G}}\right)^2$$

Where $\xi = x - \frac{t^\alpha}{(4\mu + 1 - \lambda^2)\Gamma(1+\alpha)}$,

Plugging the generalize solutions Eq. (16) into Eq. (31), we get different kinds of

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travelling wave solutions of the Eq. (2)

Case I: If $\lambda^2 - 4\mu > 0$

$$u(\xi) = -\frac{12\mu}{4\mu+1-\lambda^2} - \frac{12\lambda}{4\mu+1-\lambda^2}\left(-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{c_1\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right) + c_2\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}{c_1\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right) + c_2\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}\right)\right) + \frac{12}{4\mu+1-\lambda^2}\left(-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{c_1\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right) + c_2\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}{c_1\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right) + c_2\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}\right)\right)^2$$

Case II: If $\lambda^2 - 4\mu < 0$

$$u(\xi) = -\frac{12\mu}{4\mu+1-\lambda^2} - \frac{12\lambda}{4\mu+1-\lambda^2}\left(-\frac{\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-c_1\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right) + c_2\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)}{c_1\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right) + c_2\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)}\right)\right) + \frac{12}{4\mu+1-\lambda^2}\left(-\frac{\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-c_1\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right) + c_2\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)}{c_1\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right) + c_2\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)}\right)\right)^2$$

Case III: If $\lambda^2 - 4\mu = 0$

$$u(\xi) = -\frac{12\mu}{4\mu+1-\lambda^2} - \frac{12\lambda}{4\mu+1-\lambda^2}\left(-\frac{\lambda}{2} + \frac{c_2}{c_1+c_2\xi}\right) + \frac{12}{4\mu+1-\lambda^2}\left(-\frac{\lambda}{2} + \frac{c_2}{c_1+c_2\xi}\right)^2$$

Where $\xi = x - \frac{t^\alpha}{(4\mu+1-\lambda^2)\Gamma(1+\alpha)}$ for all above cases and c_1 and c_2 are arbitrary parameters.

Case I: if put $c_1 = 0$, then

$$u_5 = \frac{3(\lambda^2-4\mu)}{(4\mu+1-\lambda^2)}\left(1 - \coth^2\left(\frac{\sqrt{\lambda^2-4\mu}}{2}x - \frac{\sqrt{\lambda^2-4\mu}\,t^\alpha}{2(4\mu+1-\lambda^2)\Gamma(1+\alpha)}\right)\right)$$

And if $c_2 = 0$, then

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$$u_6 = \frac{3(\lambda^2 - 4\mu)}{(4\mu + 1 - \lambda^2)}\left(1 - \tanh^2\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}x - \frac{\sqrt{\lambda^2 - 4\mu}\,t^{\alpha}}{2(4\mu + 1 - \lambda^2)\Gamma(1+\alpha)}\right)\right)$$

Case II: if put $c_1 = 0$, then

$$u_7 = \frac{3(\lambda^2 - 4\mu)}{(4\mu + 1 - \lambda^2)}\left(1 + \cot^2\left(\frac{\sqrt{4\mu - \lambda^2}}{2}x - \frac{\sqrt{4\mu - \lambda^2}\,t^{\alpha}}{2(4\mu + 1 - \lambda^2)\Gamma(1+\alpha)}\right)\right)$$

And if $c_2 = 0$, then

$$u_8 = \frac{3(\lambda^2 - 4\mu)}{(4\mu + 1 - \lambda^2)}\left(1 + \tan^2\left(\frac{\sqrt{4\mu - \lambda^2}}{2}x - \frac{\sqrt{4\mu - \lambda^2}\,t^{\alpha}}{2(4\mu + 1 - \lambda^2)\Gamma(1+\alpha)}\right)\right)$$

THE TIME FRACTIONAL CAHN-ALLEN EQUATION

We consider soliton wave variable of Eq. (3) and perform the transformation Eq. (10). By plugging Eq. (10) in Eq. (3), the equation can be reduce to non-linear ODE.

(32) $-cu' - \mathrm{k}^2 u'' + u^3 - u = 0$ Suppose the solution of ODE Eq. (32) is Eq. (12). By using Eq. (14) and Eq. (15) the homogenous balance between u^3 and u'' is

$$2 + \mathrm{m} = 3\mathrm{m} \Rightarrow \mathrm{m} = 1$$

Thus from Eq. (12), we have

(33) $u(\xi) = a_0 + a_1\left(\frac{G'}{G}\right), \qquad a_1 \neq 0$

(34) $u^3 = {a_0}^3 + 3{a_0}^2 a_1\left(\frac{\mathrm{G}'}{\mathrm{G}}\right) + 3a_0{a_1}^2\left(\frac{\mathrm{G}'}{\mathrm{G}}\right)^2 + {a_1}^3\left(\frac{\mathrm{G}'}{\mathrm{G}}\right)^3$

By using Eq. (33) and Eq. (16), we get

(35) $u' = -a_1\left(\frac{\mathrm{G}'}{\mathrm{G}}\right)^2 - a_1\lambda\left(\frac{\mathrm{G}'}{\mathrm{G}}\right) - a\mu$

(36) $u'' = 2a_1\left(\frac{\mathrm{G}'}{\mathrm{G}}\right)^3 + 3a_1\lambda\left(\frac{\mathrm{G}'}{\mathrm{G}}\right)^2 + (2a_1\mu + a_1\lambda^2)\left(\frac{\mathrm{G}'}{\mathrm{G}}\right) + a_1\lambda\mu$

By plugging Eq. (33)-Eq. (36) into Eq. (32) and by equate coefficients of identical power of $\left(\frac{\mathrm{G}'}{\mathrm{G}}\right)$, we get the system of algebraic equations for a_0, a_1, k and c as follows:

$$\left(\frac{\mathrm{G}'}{\mathrm{G}}\right)^0 : a_1 c\mu - a_0 - a_1\mu\lambda\mathrm{k}^2 + {a_0}^3 = 0$$

$$\left(\frac{\mathrm{G}'}{\mathrm{G}}\right)^1 : a_1\lambda c - a_1 - 2a_1\mu\mathrm{k}^2 - a_1\lambda^2\mathrm{k}^2 + 3{a_0}^2 a_1 = 0$$

$$\left(\frac{\mathrm{G}'}{\mathrm{G}}\right)^2 : a_1 c - 3a_1\lambda\mathrm{k}^2 + 3a_0{a_1}^2 = 0$$

$$\left(\frac{\mathrm{G}'}{\mathrm{G}}\right)^3 : -2a_1\mathrm{k}^2 + {a_1}^3 = 0$$

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After solving the above algebraic equations, the constants, a_0, a_1, c and k can be achieved with the help of maple 13, we have one set of solution

(37) $a_0 = \frac{4\mu - \lambda^2 + \lambda\sqrt{\lambda^2 - 4\mu}}{2(4\mu - \lambda^2)}, a_1 = -\frac{\left(\lambda - \sqrt{\lambda^2 - 4\mu}\right)}{4\mu - \lambda^2 + \lambda\sqrt{\lambda^2 - 4\mu}}$, $\mathrm{k} = -\frac{\sqrt{2}}{2\sqrt{\lambda^2 - 4\mu}}$, $c = \frac{3}{2\sqrt{\lambda^2 - 4\mu}}$,

Where λ and μ are constant parameters, by using Eq. (37), the equation (33) can be expressed as

(38) $u(\xi) = \frac{4\mu - \lambda^2 + \lambda\sqrt{\lambda^2 - 4\mu}}{2(4\mu - \lambda^2)} - \frac{\left(\lambda - \sqrt{\lambda^2 - 4\mu}\right)}{4\mu - \lambda^2 + \lambda\sqrt{\lambda^2 - 4\mu}} \left(\frac{\mathrm{G}'}{\mathrm{G}}\right)$

Where $\xi = -\frac{\sqrt{2}}{2\sqrt{\lambda^2 - 4\mu}} x - \frac{3t^{\alpha}}{2\sqrt{\lambda^2 - 4\mu}\,\Gamma(1+\alpha)}$,

Plugging the generalize solutions Eq. (16) into Eq. (37), we get different kinds of travelling wave solutions of the Eq. (3)

Case I: If $\lambda^2 - 4\mu > 0$

$$u(\xi) = \frac{4\mu - \lambda^2 + \lambda\sqrt{\lambda^2 - 4\mu}}{2(4\mu - \lambda^2)} - \frac{(\lambda - \sqrt{\lambda^2 - 4\mu})}{4\mu - \lambda^2 + \lambda\sqrt{\lambda^2 - 4\mu}} \left(-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2 - 4\mu}}{2} \left(\frac{c_1 \sinh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi\right) + c_2 \cosh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi\right)}{c_1 \cosh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi\right) + c_2 \sinh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi\right)} \right) \right)$$

Case II: If $\lambda^2 - 4\mu < 0$

$$u(\xi) = \frac{4\mu - \lambda^2 + \lambda\sqrt{\lambda^2 - 4\mu}}{2(4\mu - \lambda^2)} - \frac{(\lambda - \sqrt{\lambda^2 - 4\mu})}{4\mu - \lambda^2 + \lambda\sqrt{\lambda^2 - 4\mu}} \left(-\frac{\lambda}{2} + \frac{\sqrt{4\mu - \lambda^2}}{2} \left(\frac{-c_1 \sin\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\xi\right) + c_2 \cos\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\xi\right)}{c_1 \cos\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\xi\right) + c_2 \sin\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\xi\right)} \right) \right)$$

Case III: If $\lambda^2 - 4\mu = 0$

$$\mathrm{u}(\xi) = \frac{4\mu - \lambda^2 + \lambda\sqrt{\lambda^2 - 4\mu}}{2(4\mu - \lambda^2)} - \frac{(\lambda - \sqrt{\lambda^2 - 4\mu})}{4\mu - \lambda^2 + \lambda\sqrt{\lambda^2 - 4\mu}} \left(-\frac{\lambda}{2} + \frac{c_2}{c_1 + c_2\xi} \right)$$

Where $\xi = -\frac{\sqrt{2}}{2\sqrt{\lambda^2 - 4\mu}} x - \frac{3t^{\alpha}}{2\sqrt{\lambda^2 - 4\mu}\,\Gamma(1+\alpha)}$ for all above cases and c_1 and c_2 are arbitrary parameters.

Case I: if put $c_1 = 0$, then

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$$u_9 = \frac{1}{2}\left(1 - \coth\left(-\frac{\sqrt{2}}{4}x - \frac{3\,t^{\alpha}}{4\Gamma(1+\alpha)}\right)\right)$$

And if $c_2 = 0$, then

$$u_{10} = \frac{1}{2}\left(1 - \tanh\left(-\frac{\sqrt{2}}{4}x - \frac{3\,t^{\alpha}}{4\Gamma(1+\alpha)}\right)\right)$$

Case II: if put $c_1 = 0$, then

$$u_{11} = \frac{1}{2}\left(1 - \cot\left(-\frac{\sqrt{2}}{4}x - \frac{3\,t^{\alpha}}{4\Gamma(1+\alpha)}\right)\right)$$

And if $c_2 = 0$, then

$$u_{12} = \frac{1}{2}\left(1 - \tan\left(-\frac{\sqrt{2}}{4}x - \frac{3\,t^{\alpha}}{4\Gamma(1+\alpha)}\right)\right)$$

NUMERICAL RESULTS AND DISCUSSION

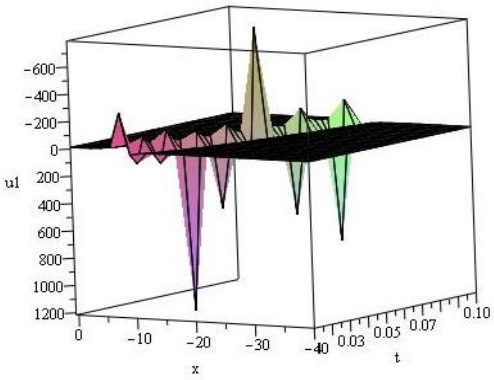


fig1: $u_1(x,t), \delta = 5, \beta = 1, \gamma = 2, \alpha = 0.4$

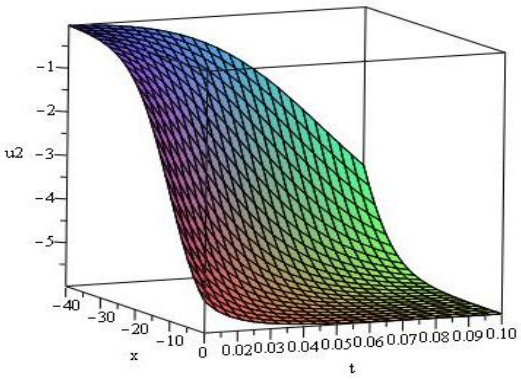


fig2: $u_2(x,t), \delta = 6, \beta = 2, \gamma = 2, \alpha = 0.6$

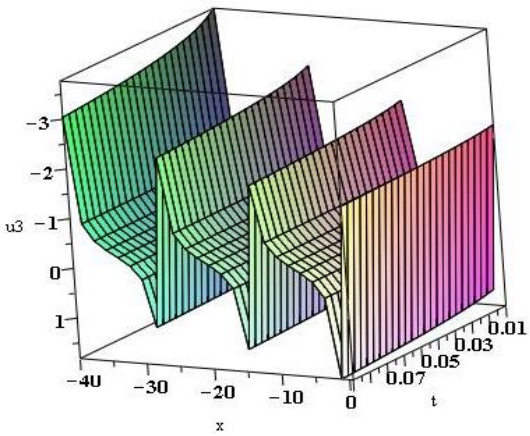


fig3: $u_3(x,t), \delta = 6, \beta = 1, \gamma = 1.5, \alpha = 0.1$

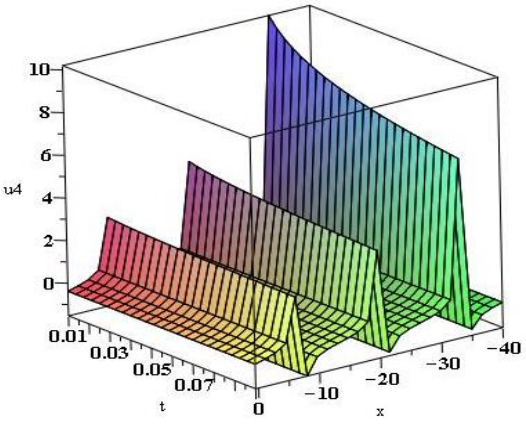


fig4: $u_4(x,t), \delta = 1, \beta = 1, \gamma = 1, \alpha = 0.3$

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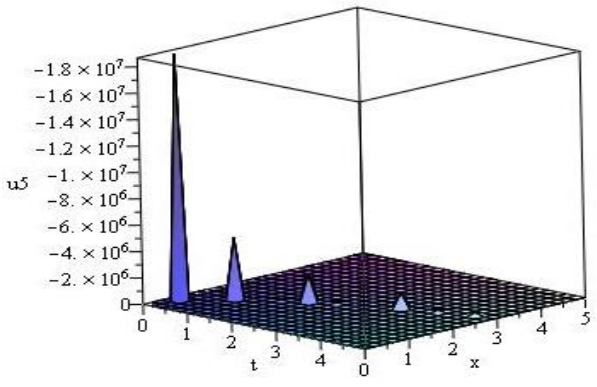


fig5: $u_5(x,t), \mu = 1, \lambda = 2.1, \alpha = 0.5$

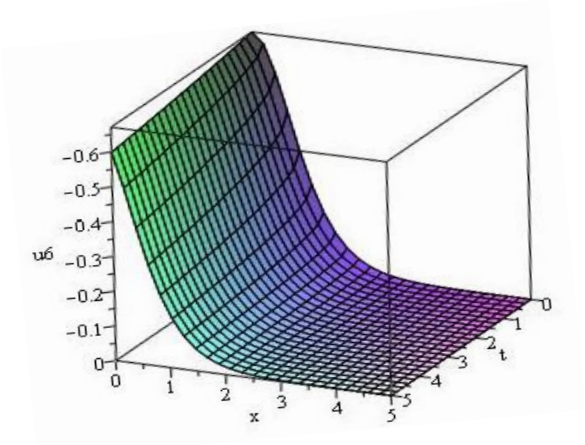


fig6: $u_6(x,t), \mu = 0.5, \lambda = 2.5, \alpha = 0.9$

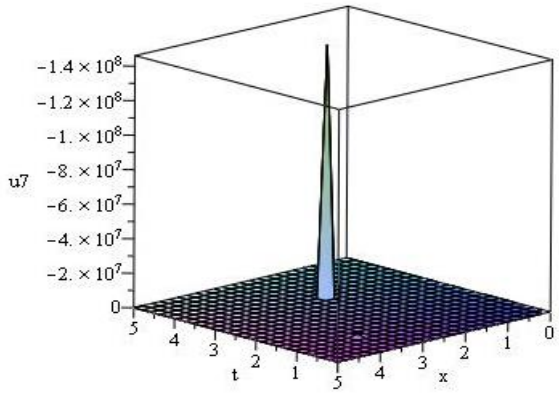


fig7: $u_7(x,t), \mu = 5, \lambda = 2.5, \alpha = 0.8$

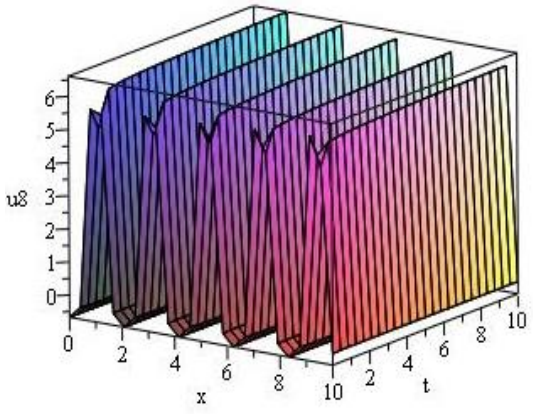


fig8: $u_8(x,t), \mu = 0.5, \lambda = 2.5, \alpha = 0.1$

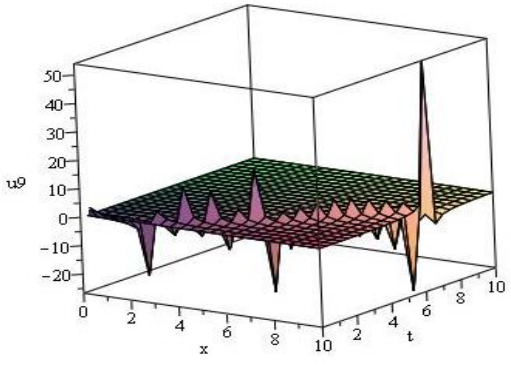


fig9: $u_9(x,t), \alpha = 0.5$

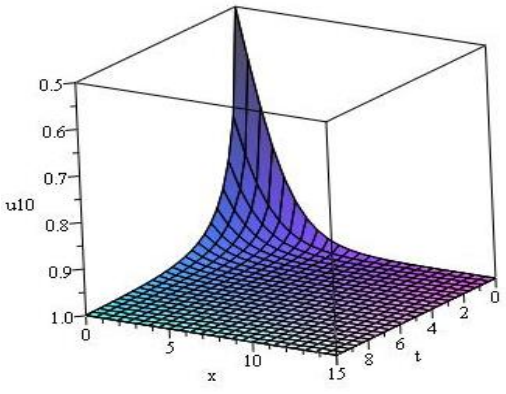


fig10: $u_{10}(x,t), \alpha = 0.6$

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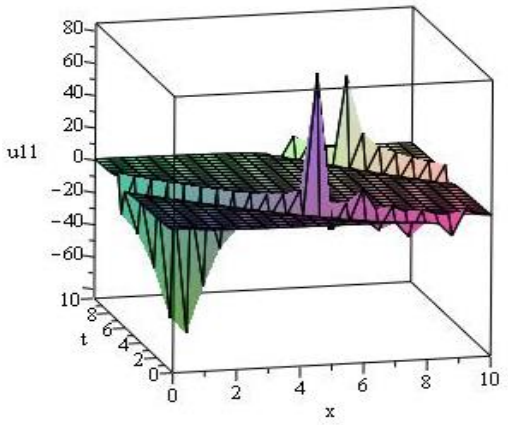


fig11: $u_{11}(x,t), \alpha = 0.7$

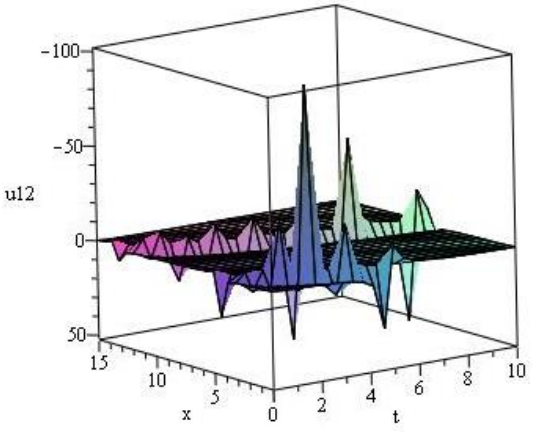


fig12: $u_{12}(x,t), \alpha = 0.4$

Through graphically illustrations, we note that the soliton is a wave that remains its shapes conserve after colliding with some other similar wave. By using non-linear evaluation equations such as benjamin-bona mahony equation, The Gardner equation and The Cahn-Allen equation We obtain the soliton wave solutions by employing randomly taken parameters values. When the velocity is positive, the soliton wave propagates in the right-direction, while a velocity is non-positive indicates leftward propagates. The velocities and amplitude are controlled by various kind of parameters. Graphically representations in the figures show relationship for various parameters values. Figures 1 to 4 describes peakon and periodic wave solution of time fractional Gardner equation by putting various parameter values. Figures 5 to 8 represent periodic wave and peakon solution of time fractional Benjamin-Bona Mahony equation by assuming different parameter values and last figures 9 to 12 describe peakon and periodic wave solution of time-fractional Cahn-Allen equation by various values of parameter. Specially, the soliton wave solutions are compaction solutions, peakon solutions and other solutions have been found for such physical problems. In a nutshell, we obtained similar soliton wave solutions for various parameters values, which entirely show that final solution is not always dependent on these parameters values effectively. Therefore, we can assume random parameters values as input into our solutions. The results are achieved by $\left(\frac{G'}{G}\right)$-expansion approach. Graphically observations shows the accuracy of the presented approach.

CONCLUSIONS

In this article, we have examined three well-known non-linear fractional differential equations named benjamin-bona mahony equation, The Gardner equation and The Cahn-Allen equation. For the purpose to construct the travelling wave solutions of non-linear FDE's, we used $\left(\frac{G'}{G}\right)$- expansion method. This approach converts the non-linear FDE's to another non-linear ODE's by using suitable transformations. By comparing homogeneity balance, we convert the non-linear ODE to the system of non-linear algebraic equations. The solution set of nonlinear algebraic system is

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obtained by using Maple 13. The obtained travelling wave solutions are important to understand the physical behavior of the non-linear FDE's and can be used in many areas of mathematical physics. The graphical illustrations of the obtained solution is also presented by using Maple 13. Finally, it is concluded that the $\left(\frac{G'}{G}\right)$-expansion method is very effective and reliable to be used to constructs the closed form solutions of many physical problems.

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