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A Hybrid Complex Fermatean Fuzzy Hypersoft Set-Based MADM Model for International Trade Partner Selection

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ABSTRACT

In today's globalized era, businesses strive to expand internationally but face challenges in selecting the most suitable country for partnership due to variations in political stability, economic conditions, trading capacity, volume, and policies. To address this, an effective multi-attribute decision-making (MADM) approach is required to evaluate multiple parameters. This article proposes an algorithm based on a hybrid structure of the Complex Fermatean Fuzzy Hypersoft Set (CFFHSS) that incorporates two efficient distance measures to identify the best partner country for international trade. The proposed algorithm is applied to a case study of a textile company seeking to expand its business internationally by selecting the best trading partner among Japan, Germany, the UAE, and Canada, considering economic stability and market accessibility. Statistical analysis confirms the significance of the proposed method, followed by a discussion of research findings, limitations, and future directions.

Keywords: Multi-Attribute Problems, Decision Methods, Distance Measures, Global Trading, Fuzzy Systems, Statistical Analysis,

Introduction

In today's complex and advanced trading era, decisions made within trade policies affect the economy very rapidly and significantly. Sudden changes in global trade patterns, shifts in trends, rise in policies of protectionists, trading disputes, political instabilities and technological shifts have collectively made a complex and unpredictable environment for policymakers. Therefore, creating and designing effective techniques or strategies for international trade has become a challenging task, requiring innovative approaches that can handle the complexities of the global market.

Online ISSN

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A complex and uncertain environment has been created for trade policymakers by the quick changes in global trade patterns, as well as the rise in protectionism and trade disputes. Formulating strategies that work for international trade has become a difficult task that requires innovative thinking that can deal the complexities of global trade.

The complexities and vagueness intrinsic in global trade are often ignored by conventional approaches to international trade policy decisions, which often rely on oversimplified assumptions. In order to overcome this constraint, this research proposal will use Complex Fermatean Fuzzy Hypersoft Set (CFFHSS) to create a unique framework for creating strategies for inter-national trade interactions. Modeling complex trade interactions and identifying the best trade tactics can be made possible by CFFHSS, a potent tool for managing complex, ambiguous, and fuzzy data. In Soft Set theory an element of a subset of a set of parameters is taken along with related subset of universal set as ordered paired. This process gives subsets of universal set which are related to specific parameters. But it doesn't deal with subparameters of parameters, to resolve this problem extension of soft set is needed. This extension is Hypersoft Set. This extension can be used in Complex Fermatean Fuzzy Soft Set (CFFSS) to get CFFHSS. This will give accurate and fast results than CFFSS. Zadeh [1] extended the concept of traditional sets by associating a specific membership value with each element, which belongs to the closed interval of zero and one in 1965, which he named a fuzzy set. After that, Atanassov [2] gave the concept that along with membership value, non-membership value is also necessary. To fulfill this purpose, he presented an extension of fuzzy sets, which he named intuitionistic fuzzy sets. Both these membership and non-membership values belong to the zero-one interval, and their sum must be less than one. Abbasov [3] presented an extension of IFS, saying that if the sum of membership and non-membership values is not less than one, then the sum of their squares can be less than one. They named this extension Pythagorean fuzzy sets. Senapati and Yager [4] said that if the sum of squares of membership and non-membership values is not less than or equal to one, then the sum of their cubes can be less than or equal to one. They named this extension Fermatean fuzzy sets.

In fuzzy sets, the data that is written is in a single dimension, but to write double dimension data, a complex plane is required. To overcome this limitation, Ramot et al. [5] introduced the idea of complex fuzzy sets. Alkouri and Salleh [6] introduced the concept of complex intuitionistic fuzzy sets, which includes both membership and non-membership values. After that, Ullah et al. [7] presented an extension of CIFS, which he named complex Pythagorean fuzzy sets, where the sum of the squares of the amplitude terms and the sum of the squares of the phase terms of membership and non-membership values are each less than one. After that, Chinnadurai et al. [8] proposed the idea of complex Fermatean fuzzy sets, where the sum of the cubes of the amplitude terms and the sum of the cubes of the phase terms of membership and non-membership values are each less than one.

After that, Molodtsov [9] introduced the concept of soft set theory, where a soft set involves a mapping from a collection of parameters to the power set of the universal set. Then, Cagman et al. [10] introduced fuzzy soft sets, an advancement of soft sets. Smarandache [17] introduced the concept of hypersoft sets, an extension of soft sets that can handle sub-parameters along

Online ISSN

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3007-3189

<http://amresearchreview.com/index.php/Journal/about>

with parameters. Later, researchers further developed the idea of hypersoft sets. To deal with high uncertainty, advancements were made by extending hypersoft sets. After that, Rahman [22] combined fuzzy hypersoft set structures with the complex plane, introducing new structures that can handle uncertainties with higher accuracy and provide better results. These structures include complex fuzzy hypersoft sets, complex intuitionistic fuzzy hypersoft sets, and complex Pythagorean fuzzy hypersoft sets, which are based on the foundation of complex fermatean fuzzy hypersoft sets.

Characteristics of Complex Fermatean Fuzzy Hypersoft Set

This section provides a detailed explanation of the structure of Complex Fermatean Fuzzy Hypersoft Sets (CFFHSS). It begins with basic definitions, followed by a definition of complex Fermatean fuzzy hypersoft sets, accompanied by illustrative examples. Additionally, two distance measures are defined and clarified with examples to facilitate understanding.

Fundamental knowledge

Before understanding the structure of Complex Fermatean Fuzzy Hypersoft Sets (CFFHSS), it is essential to grasp the concept of the following fundamental definitions.

Definition 1.1. Let $\tilde{U}$ be the universal set then the Complex Fuzzy Set(CFS) defined on $\tilde{U}$ can be written as:

$$CF = \{ \langle x, (\mu(x)) \rangle | x \in \tilde{U} \}$$

where $\mu(x) = \eta e^{2\pi i \zeta}$ with the condition $\eta(x), \zeta(x) \in [0, 1]$.

Definition 1.2. Let $\tilde{U}$ be universal set and $CF = \{ \langle x, \mu(x) \rangle | x \in \tilde{U}, \mu(x) = \eta e^{2\pi i \zeta} \}$ be a

Complex fuzzy set(CFS) defined on $\tilde{U}$ then Complex Fuzzy Soft Set(CFSS) is expressed in the following way:

$$CFS = \{ \langle e, F(e) \rangle | e \in A \},$$

here $F(e)$ is the Complex Fuzzy Subset of $\tilde{U}$.

Definition 1.3. Let $\beta = \{ \beta_1, \beta_2, \dots, \beta_n \} (n \geq 1)$ describes the set of attributes/parameters, and ϵ_i is set of corresponding sub-attributes/sub-parameters of β_i , respectively having $\epsilon_i \cap \epsilon_j = \emptyset$ for $n \geq 1$ for all $i, j \in N$, $i \neq j$. suppose $\epsilon_1 \times \epsilon_2 \times \dots \times \epsilon_n = \sigma = \{ (m_{1p} \times m_{2q} \times \dots \times m_{nx} : \text{where } 1 \leq p, q \wedge p \leq a, q \leq b, 1 \leq x \leq c \wedge a, b, c \in N) \}$ represents set of multi-attributes then

F_σ is called Complex Fuzzy Hypersoft Set (CFHSS) over $\tilde{U}$ and it can be expressed by:

$$F_\sigma = \{ \langle \tilde{m}, F_\sigma(\tilde{m}) \rangle | \tilde{m} \in \sigma \wedge F_\sigma(\tilde{m}) \in (CFS(\tilde{U}))^n \},$$

Online ISSN

3007-3197

Print ISSN

3007-3189

<http://amresearchreview.com/index.php/Journal/about>

here $\tilde{U}$ is universal set and $F_{\sigma} : \sigma \rightarrow CFS(\tilde{U})$, that is, $F_{\sigma}(\tilde{m}) = \{(\rho, \mu_{\rho}) : \rho \in \tilde{U}\}$ in which μ_{ρ}

is membership function of ρ such as $\mu_{\rho} = \eta_F(\rho)e^{2\pi i\zeta_F(\rho)}$ where $\eta_F(\rho), \zeta_F(\rho) \in [0, 1]$ and

Definition 1.4. Let $\beta = \{\beta_1, \beta_2, \dots, \beta_n\} (n \geq 1)$ describes the set of attributes/parameters, and ϵ_i is set of corresponding sub-attributes/sub-parameters of β_i , respectively having $\epsilon_i \cap \epsilon_j = \emptyset$ for $n \geq 1$ for all $i, j \in N$, $i \neq j$. suppose $\epsilon_1 \times \epsilon_2 \times \dots \times \epsilon_n = \sigma = \{(m_{1p} \times m_{2q} \times \dots \times m_{nx} :$

where $1 \leq p, q \wedge p \leq a, q \leq b, 1 \leq x \leq c \wedge a, b, c \in N\}$ represents set of multi-attributes then F_{σ} is called Complex Pythagorean Fuzzy Hypersoft Set (CPFHSS) over $\tilde{U}$ and it can be expressed by:

$$F_{\sigma} = \langle \tilde{m}, F_{\sigma}(\tilde{m}) \rangle | \tilde{m} \in \sigma \wedge F_{\sigma}(\tilde{m}) \in (CPFS(\tilde{U}))',$$

where as $\tilde{U}$ is universal and $F_{\sigma} : \sigma \rightarrow CPFS(\tilde{U})$, that is, $F_{\sigma}(\tilde{m}) = \{(\rho, (\mu_{\rho}, \nu_{\rho})) : \rho \in \tilde{U}\}$ in

which μ_{ρ}, ν_{ρ} are membership and non membership functions of ρ respectively such as $\mu_{\rho} = \eta_{1F}(\rho)e^{2\pi i\zeta_{1F}(\rho)}, \nu_{\rho} = \eta_{2F}(\rho)e^{2\pi i\zeta_{2F}(\rho)}$ where $\eta_{1F}(\rho), \eta_{2F}(\rho) \in [0, 1]$ and $\zeta_{1F}(\rho), \zeta_{2F}(\rho) \in [0, 1]$

also $(\eta_{1F}(\rho))^2 + (\eta_{2F}(\rho))^2 \in [0, 1]$ and $(\zeta_{1F}(\rho))^2 + (\zeta_{2F}(\rho))^2 \in [0, 1]$.

Complex fermatean fuzzy hypersoft set and its characteristics

Now, Complex Fermatean Fuzzy Hypersoft Sets will be defined, and examples will be provided to facilitate easy understanding.

Definition 1.5. Let $\beta = \{\beta_1, \beta_2, \dots, \beta_n\} (n \geq 1)$ describes the set of attributes/parameters, and ϵ_i is set of corresponding sub-attributes/sub-parameters of β_i , respectively having $\epsilon_i \cap \epsilon_j = \emptyset$ for $n \geq 1$ for all $i, j \in N$, $i \neq j$. suppose $\epsilon_1 \times \epsilon_2 \times \dots \times \epsilon_n = \sigma = \{(m_{1p} \times m_{2q} \times \dots \times m_{nx} :$ where $1 \leq p, q \wedge p \leq a, q \leq b, 1 \leq x \leq c \wedge a, b, c \in N\}$ represents set of multi-attributes

then F_{σ} is called Complex Fermatean Fuzzy Hypersoft Set (CFFHSS) over $\tilde{U}$ and it can be

expressed by:

$$F_{\sigma} = \langle \tilde{m}, F_{\sigma}(\tilde{m}) \rangle | \tilde{m} \in \sigma \wedge F_{\sigma}(\tilde{m}) \in (CFFS(\tilde{U}))',$$

here $\tilde{U}$ is a universal set and $F_{\sigma} : \sigma \rightarrow CFFS(\tilde{U})$, that is, $F_{\sigma}(\tilde{m}) = \{(\rho, (\mu_{\rho}, \nu_{\rho})) | \rho \in \tilde{U}\}$ in

which μ_{ρ}, ν_{ρ} are membership and non membership functions of ρ respectively such as $\mu_{\rho} = \eta_{1F}(\rho)e^{2\pi i\zeta_{1F}(\rho)}, \nu_{\rho} = \eta_{2F}(\rho)e^{2\pi i\zeta_{2F}(\rho)}$ where $\eta_{1F}(\rho), \eta_{2F}(\rho) \in [0, 1]$ and $\zeta_{1F}(\rho), \zeta_{2F}(\rho) \in [0, 1]$

Online ISSN

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Print ISSN

3007-3189

<http://amresearchreview.com/index.php/Journal/about>

also $(\eta_{1F}(\rho))^3 + (\eta_{2F}(\rho))^3 \in [0, 1]$ and $(\zeta_{1F}(\rho))^3 + (\zeta_{2F}(\rho))^3 \in [0, 1]$.

Example 1.6.

$$\tilde{U} = \{V \text{ ivo, Oppo}\}$$

$$E_1 = \{\text{camera, Memory}\}$$

$$E_2 = \{\text{Display, Battery}\}$$

$$\text{CPFHSS} =$$

$$\begin{aligned} & \{((\text{Camera, Display}), \{(V \text{ ivo}, 0.75e^{2\pi i(0.6)}, 0.75e^{2\pi i(0.9)}), (\text{Oppo}, 0.99e^{2\pi i(0.98)}, 0.2e^{2\pi i(0.25)})\})\} \\ & , ((\text{Camera, Battery}), \{(V \text{ ivo}, 0.8e^{2\pi i(0.3)}, 0.4e^{2\pi i(0.1)}), (\text{Oppo}, 0.5e^{2\pi i(0.2)}, 0.8e^{2\pi i(0.6)})\})\} \\ & , ((\text{Memory, Display}), \{(V \text{ ivo}, 0.7e^{2\pi i(0.6)}, 0.8e^{2\pi i(0.85)}), (\text{Oppo}, 0.7e^{2\pi i(0.1)}, 0.5e^{2\pi i(0.7)})\})\} \\ & , ((\text{Memory, Battery}), \{(V \text{ ivo}, 0.9e^{2\pi i(0.3)}, 0.2e^{2\pi i(0.6)}), (\text{Oppo}, 0.4e^{2\pi i(0.4)}, 0.7e^{2\pi i(0.5)})\})\} \end{aligned}$$

Methodology

This section provided the MADM approach based on CFFHSS and then a case study which is solved by proposed model.

Algorithmic approach based on distance measure of CFFHSS

The proposed multi parameters decision making approach using distance measures of CFFHSS has the following steps:

Construct CFFHSS on the selection set $\tilde{U} = \{\rho_1, \rho_2, \rho_3, \dots, \rho_n\}$ by using set of multi parameters $\sigma = \{m_1, m_2, m_3, \dots, m_m\}$ with the help of expert opinion, e.g.

$$(\rho_1, (\eta_{m_1 \rho_1} e^{2\pi i \zeta_{m_1 \rho_1}}, \tilde{\eta}_{m_1 \rho_1} e^{2\pi i \tilde{\zeta}_{m_1 \rho_1}})),$$

$$F_{\sigma}(m_1) = \left\{ \begin{aligned} & (\rho_1, (\eta_{m_1 \rho_1} e^{2\pi i \zeta_{m_1 \rho_1}}, \tilde{\eta}_{m_1 \rho_1} e^{2\pi i \tilde{\zeta}_{m_1 \rho_1}})), \\ & (\rho_2, (\eta_{m_2 \rho_2} e^{2\pi i \zeta_{m_2 \rho_2}}, \tilde{\eta}_{m_2 \rho_2} e^{2\pi i \tilde{\zeta}_{m_2 \rho_2}})), \\ & (\rho_3, (\eta_{m_3 \rho_3} e^{2\pi i \zeta_{m_3 \rho_3}}, \tilde{\eta}_{m_3 \rho_3} e^{2\pi i \tilde{\zeta}_{m_3 \rho_3}})), \\ & \vdots \\ & (\rho_n, (\eta_{m_n \rho_n} e^{2\pi i \zeta_{m_n \rho_n}}, \tilde{\eta}_{m_n \rho_n} e^{2\pi i \tilde{\zeta}_{m_n \rho_n}})), \end{aligned} \right\}$$

□
□

Rationalize the CFFHSS for each element (ρ_i for $i = 1, 2, 3, \dots, n$) of selection set as given in Table 1:

CFFHSS DV	m_1	m_2	$\dots$	m_m
ρ_i	$\eta_{m_1 \rho_i} e^{2\pi i \zeta_{m_1 \rho_i}}$ $\tilde{\eta}_{m_1 \rho_i} e^{2\pi i \tilde{\zeta}_{m_1 \rho_i}}$	$\eta_{m_2 \rho_i} e^{2\pi i \zeta_{m_2 \rho_i}}$ $\tilde{\eta}_{m_2 \rho_i} e^{2\pi i \tilde{\zeta}_{m_2 \rho_i}}$	$\dots$	$\eta_{m_m \rho_i} e^{2\pi i \zeta_{m_m \rho_i}}$ $\tilde{\eta}_{m_m \rho_i} e^{2\pi i \tilde{\zeta}_{m_m \rho_i}}$

TABLE 1. Rationalized elements of CFFHSS for all ρ_i

Online ISSN

3007-3197

Print ISSN

3007-3189

<http://amresearchreview.com/index.php/Journal/about>

Create the ideal CFFHSS matrix by choosing maximum $\eta_{\rho j}$ from all $\eta_{\rho i}$ m_i and minimum $\tilde{\eta}_{\rho k}$ from all $\tilde{\eta}_{\rho i}$ for each m_i as given in Table 2.

CFFHSS DV	m_1	m_2	$\dots$	m_m
ρ	$\eta_{m_1\rho_j} e^{2\pi i \zeta_{m_1\rho_j}}$	$\eta_{m_2\rho_j} e^{2\pi i \zeta_{m_2\rho_j}}$	$\dots$	$\eta_{m_m\rho_j} e^{2\pi i \zeta_{m_m\rho_j}}$
	$\tilde{\eta}_{m_1\rho_k} e^{2\pi i \tilde{\zeta}_{m_1\rho_k}}$	$\tilde{\eta}_{m_2\rho_k} e^{2\pi i \tilde{\zeta}_{m_2\rho_k}}$	$\dots$	$\tilde{\eta}_{m_m\rho_k} e^{2\pi i \tilde{\zeta}_{m_m\rho_k}}$

TABLE 2. The ideal ρ matrix

Find the distance measures $d_1(\rho, \rho_i)$, $d_2(\rho, \rho_i)$ by using formulas given as:

$$d_1(F_\sigma, G_\sigma) = \left[\frac{1}{pq} \sum_{i=1}^p \sum_{j=1}^q \left(|\eta_{(m_i\rho_j)F} - \eta_{(m_i\rho_j)G}|^2 + |\tilde{\eta}_{(m_i\rho_j)F} - \tilde{\eta}_{(m_i\rho_j)G}|^2 \right) + \frac{1}{2\pi} \left[|\zeta_{(m_i\rho_j)F} - \zeta_{(m_i\rho_j)G}|^2 + |\tilde{\zeta}_{(m_i\rho_j)F} - \tilde{\zeta}_{(m_i\rho_j)G}|^2 \right] \right]^{\frac{1}{2}}$$

and

$$d_2(F_\sigma, G_\sigma) = \frac{1}{pq} \sum_{i=1}^p \sum_{j=1}^q \left(\left[|\eta_{(m_i\rho_j)F}^3 - \eta_{(m_i\rho_j)G}^3| + |\tilde{\eta}_{(m_i\rho_j)F}^3 - \tilde{\eta}_{(m_i\rho_j)G}^3| \right] + \frac{1}{2\pi} \left[|\zeta_{(m_i\rho_j)F}^3 - \zeta_{(m_i\rho_j)G}^3| + |\tilde{\zeta}_{(m_i\rho_j)F}^3 - \tilde{\zeta}_{(m_i\rho_j)G}^3| \right] \right)$$

Calculate the mean of distance measures by d_1+d_2

Rank the distance measures from highest to least.

Choose the element ρ from selection set $\tilde{U}_{best}$ with least distance measure, this is the selection from all the given options.

Case Study

A textile manufacturing company is planning to expand its international business by ex-

Online ISSN

3007-3197

Print ISSN

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<http://amresearchreview.com/index.php/Journal/about>

porting its premium fabric products to global markets. The managements goal is to identify the most reliable export destination that ensures both high trade potential and long-term stability. The decision to choose the best export market involves multiple attributes, requiring the analysis of both qualitative and quantitative factors. To achieve this, the companys manager consulted a panel of trade and economic experts. Each expert provided their assessments in the form of membership and non-membership degrees for various potential trade partners under specific parameters and sub-parameters. For this purpose, the evaluation was performed using the Complex Fermatean Fuzzy Hypersoft Set (CFFHSS) model. This approach helps to represent uncertainty and expert hesitation when analyzing international trade factors. Four countries were selected as potential trading destinations because of their strong textile demand, trade openness, and business relations with developing economies.

Hence, the universal set of alternatives is defined as:

$\tilde{U} = \{\rho_1 = \text{japan}, \rho_2 = \text{UAE}, \rho_3 = \text{Germany}, \rho_4 = \text{Canada}\}$

This includes two key trade factors:

$\beta = \{\beta_1 = \text{market accessibility}, \beta_2 = \text{economic stability}\}$

Each parameter has two sub-parameters as:

$\beta_1 = \{\text{high accessibility} = m_{11}, \text{moderate accessibility} = m_{12}\}$ $\beta_2 = \{\text{stable economy} = m_{21}, \text{volatile economy} = m_{22}\}$

Now collection of multi parameters will be

$\sigma = \{m_1 = (m_{11}, m_{21}), m_2 = (m_{11}, m_{22}), m_3 = (m_{12}, m_{21}), m_4 = (m_{12}, m_{22})\}$

The decision of the experts, in the form of CFFHSS for the selction of a country from the set $\tilde{U} = \{\rho_1, \rho_2, \rho_3, \rho_4\}$ by using multi parameters m_1, m_2, m_3 and m_4 is given as

Online ISSN

3007-3197

Print ISSN

3007-3189

<http://amresearchreview.com/index.php/Journal/about>

$$F_{\sigma}(m_1) = \begin{pmatrix} (\rho_1, (0.90e^{2\pi i 0.85}, 0.60e^{2\pi i 0.65})) \\ (\rho_2, (0.70e^{2\pi i 0.65}, 0.40e^{2\pi i 0.45})) \\ (\rho_3, (0.75e^{2\pi i 0.90}, 0.40e^{2\pi i 0.20})) \\ (\rho_4, (0.60e^{2\pi i 0.80}, 0.45e^{2\pi i 0.30})) \end{pmatrix}$$

$$F_{\sigma}(m_2) = \begin{pmatrix} (\rho_1, (0.70e^{2\pi i 0.85}, 0.25e^{2\pi i 0.25})) \\ (\rho_2, (0.75e^{2\pi i 0.78}, 0.30e^{2\pi i 0.32})) \\ (\rho_3, (0.80e^{2\pi i 0.90}, 0.35e^{2\pi i 0.25})) \\ (\rho_4, (0.65e^{2\pi i 0.85}, 0.45e^{2\pi i 0.30})) \end{pmatrix}$$

$$F_{\sigma}(m_3) = \begin{pmatrix} (\rho_1, (0.88e^{2\pi i 0.75}, 0.32e^{2\pi i 0.40})) \\ (\rho_2, (0.70e^{2\pi i 0.92}, 0.45e^{2\pi i 0.20})) \\ (\rho_3, (0.80e^{2\pi i 0.68}, 0.35e^{2\pi i 0.46})) \\ (\rho_4, (0.85e^{2\pi i 0.73}, 0.28e^{2\pi i 0.40})) \end{pmatrix}$$

$$F_{\sigma}(m_4) = \begin{pmatrix} (\rho_1, (0.90e^{2\pi i 0.83}, 0.25e^{2\pi i 0.33})) \\ (\rho_2, (0.78e^{2\pi i 0.66}, 0.37e^{2\pi i 0.47})) \\ (\rho_3, (0.87e^{2\pi i 0.79}, 0.27e^{2\pi i 0.36})) \\ (\rho_4, (0.74e^{2\pi i 0.82}, 0.38e^{2\pi i 0.30})) \end{pmatrix}$$

Rationalized CFFHSS for each ρ_i is given in Table 3.

Online ISSN

3007-3197

Print ISSN

3007-3189

<http://amresearchreview.com/index.php/Journal/about>

CFFHSS DV	m_1	m_2	m_3	m_4
ρ_1	$0.90^{0.85}$	$0.70^{0.85}$	$0.88^{0.75}$	$0.90^{0.83}$
	$0.60^{0.65}$	$0.25^{0.25}$	$0.32^{0.40}$	$0.25^{0.33}$
ρ_2	$0.75^{0.65}$	$0.75^{0.78}$	$0.70^{0.92}$	$0.78^{0.66}$
	$0.40^{0.45}$	$0.30^{0.32}$	$0.45^{0.20}$	$0.38^{0.30}$
ρ_3	$0.75^{0.90}$	$0.80^{0.90}$	$0.80^{0.68}$	$0.87^{0.79}$
	$0.40^{0.20}$	$0.35^{0.25}$	$0.35^{0.46}$	$0.27^{0.36}$
ρ_4	$0.60^{0.80}$	$0.65^{0.85}$	$0.85^{0.73}$	$0.74^{0.82}$
	$0.45^{0.30}$	$0.45^{0.30}$	$0.28^{0.40}$	$0.38^{0.30}$

TABLE 3. The Rationalized form of CFFHSS for each ρ

The Ideal CFFHSS matrix value is evaluated and shown in Table Table 4.

The calculated distance measures $d_1(\rho, \rho_i)$ and $d_2(\rho, \rho_i)$ by using generalized Euclidean distance measures ?? and fermatean distance measure ?? respectively, are shown in Table 5.

The mean of all the distance measures is given in Table 6.

Online ISSN

3007-3197

Print ISSN

3007-3189

<http://amresearchreview.com/index.php/Journal/about>

CFFHSS DV	m_1	m_2	m_3	m_4
ρ	0.90 ^{0.85}	0.80 ^{0.90}	0.88 ^{0.75}	0.90 ^{0.83}
	0.40 ^{0.20}	0.25 ^{0.25}	0.28 ^{0.40}	0.25 ^{0.33}

TABLE 4. The ideal CFFHSS matrix

Distance Measures	(ρ, ρ_1)	(ρ, ρ_2)	(ρ, ρ_3)	(ρ, ρ_4)
$d_1(\rho, \rho_i)$	0.073	0.098	0.054	0.112
$d_2(\rho, \rho_i)$	0.025	0.083	0.041	0.083

TABLE 5. Distance measures between available CFFHSS matrices and ideal matrix

$d(\rho, \rho_i)$	Japan	UAE	Germany	Canada
ρ	0.049	0.091	0.048	0.098

TABLE 6. Average of CFFHSS DMs by using arithmetic mean

Ranking of distance measures is as:

$$d(\rho, \rho_3) \leq d(\rho, \rho_1) \leq d(\rho, \rho_2) \leq d(\rho, \rho_4)$$

As ρ_1 has least distance measure with the ideal alternative so Germany is the best selection from available options.

Statistical Analysis

In this test, let's will check the statistical significance of the proposed DMs. We claimed that there is a statistical significant difference between the both proposed DMs. Now we will prove our claim by using Paired T-Test.

First of all calculate the differences of both distance values of Table 5 shown in Table 7 :

Distance Measures	(ρ, ρ_1)	(ρ, ρ_2)	(ρ, ρ_3)	(ρ, ρ_4)
$d_1(\rho, \rho_i)$	0.073	0.098	0.054	0.112
$d_2(\rho, \rho_i)$	0.025	0.083	0.041	0.083
$d = d_1 - d_2$	0.048	0.015	0.013	0.029

TABLE 7. Differences of both distance values

Online ISSN

3007-3197

Print ISSN

3007-3189

<http://amresearchreview.com/index.php/Journal/about>

Calculate the mean $\bar{d}$ of the differences:

$$\bar{d} = \frac{0.048 + 0.015 + 0.013 + 0.029}{4} = 0.06$$

Calculate the Squared Deviations $(d_i - \bar{d})^2$ for Standard Deviation S_d of the differences show in Table 8:

Distance Measures	Japan	UAE	Germany	Canada
$d = d_1 - d_2$	0.048	0.015	0.013	0.029
Squared Deviations	= 0.000144	0.002025	0.002209	0.000961

TABLE 8. Squared Deviation

Compute the sum of these squared deviations:

$$(d_i - \bar{d})^2 = 0.000144 + 0.002025 + 0.002209 + 0.000961 = 0.005339$$

now divide this sum by $n - 1 = 4 - 1 = 3$ and take square root to find standard deviation S_d :

$$S_d = \sqrt{\frac{0.005339}{3}} = 0.001779$$

find t-statistic:

$$t = \frac{\bar{d}}{S_d} = \frac{0.06}{0.001779} = 674.536$$

calculate p value:

$$t = \frac{\bar{d}}{S_d} = \frac{0.06}{0.001779} = 674.536$$

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use the t-distribution with $n - 1 = 3$ degrees of freedom. Look up $t = 674.536$ in a t-distribution table for $df = 3$, we get $P = 0.00000000325$

now

$p - \text{value} = 2 \times P(T > 674.536) = 2 \times 0.00000000325 = 0.0000000065$

since $p - \text{value} = 0.0000000065 \lll 0.05$, the result indicates that there is an extremely significant statistical difference between the two distance measures d_1 and d_2 .

Conclusion

This article introduces a new framework of complex Fermatean fuzzy hypersoft sets, as existing structures such as FFHSS, CPFHSS, PFHSS, CIFHSS, IFHSS, CFHSS, and FHSS have significant deficiencies. These structures have been critically examined, and their limitations outlined. A new definition of complex Fermatean fuzzy hypersoft sets has been introduced to overcome these limitations. Additionally, two new distance measures have been proposed, one based on Euclidean distance and the other on a similar environment-based approach. These distance measures were applied to select the best training partner. A case study was conducted to select the best partner country for international trading for a textile company. A statistical analysis of the case study was then performed, highlighting its statistical significance. Finally, the advantages of the proposed framework were highlighted.

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